

## *Kant's Treatment of the Mathematical Antinomies in the First Critique and in the Prolegomena: A Comparison\**

ALBERTO VANZO

*Department of Philosophy,*

*University of Padova – University of Venezia 'Ca' Foscari'*

*This paper deals with an apparent contradiction between Kant's account and solution of the mathematical antinomies of pure speculative reason in the Critique of Pure Reason and in the Prolegomena. In the first Critique, Kant claims that the theses are affirmative judgments, of the form 'A is B', and the antitheses are infinite judgments, of the form 'A is non-B'. The theses and the antitheses are contradictorily opposed (i.e., the one true and the other false) and their proofs are valid only if a certain condition takes place, that is, if the world has a determinate magnitude. Otherwise, both are false and their proofs are wrong. Given transcendental realism, this condition takes place and the mathematical antinomies arise. Given transcendental idealism, this condition does not take place, the theses and the antitheses are false, and the mathematical antinomies disappear. At a first glance, according to the Prolegomena the theses are affirmative judgments and the antitheses are not infinite judgments, but negative judgments, of the form 'A is not B'. Transcendental idealism granted, the subject common to theses and antitheses, namely, the concept of 'world', is inconsistent. Both judgments are false by the rule 'non entis nulla sunt praedicata' and the antinomies do not take place. These accounts seem to be incompatible with each other. Are the antitheses infinite or negative judgments? Are the antinomies solved because the world does not have a determinate magnitude, or because its notion is inconsistent?*

*The paper argues that the contrast between the first Critique and the Prolegomena is only apparent. It depends on an error in the most natural interpretation of the paragraphs on the mathematical antinomies in the Prolegomena. The text of the Prolegomena gives the reader the impression, but it does not explicitly claim, that the antitheses are negative judgments, rather than infinite ones. In that case it is possible to hold that, for the Prolegomena, the antitheses are infinite judgments, as they are for the Critique, and they are contradictorily opposed to each other only if the world has a determinate magnitude. In addition to what is explained in*

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the Critique, the Prolegomena make clear that both theses and antitheses have an inconsistent subject concept. On this reading, the Critique and the Prolegomena are not in contrast with each other. They rather complete each other, giving the reader a fuller comprehension of the solution of the mathematical antinomies. The theses and the antitheses are false because their subject is inconsistent, as the Prolegomena maintain. Their proofs are wrong because the world does not have a determinate magnitude, as the Critique claims.

### Introduction<sup>1</sup>

The solution of the mathematical antinomies of pure speculative reason plays a central role for the whole Critical philosophy. It provides a way out of skepticism, it is the basis for the indirect proof of transcendental idealism, and it is indispensable in persuading man to accept the 'Copernican revolution' of Critical philosophy. From a systematical point of view, the treatment and solution of the antinomies in the first *Critique* is a model for the solution of the antinomies which arise in moral philosophy, aesthetics and the study of living beings.

A crucial step toward the solution of the antinomies is the analysis of the relationship between theses and antitheses. As it is well-known, every antinomy is made up of two judgments which are opposed to each other, but proven by seemingly valid arguments and, therefore, purportedly both true. If two judgments are opposed, the truth of each one implies the falsity of the other, so they cannot both be true.<sup>2</sup> The "contradiction of reason with itself" consists of the claim of truth of both opposed judgments. To solve this contradiction, Kant demonstrates that (1) theses and antitheses are not both true (in the mathematical antinomies), or that they may both be true, but they are not opposed to each other (in the dynamical ones), and that (2) the proofs of false theses and antitheses are wrong. Kant constructs both parts of this solution on the analysis of the relationship between theses and antitheses. If it were not correct, the solution of the antinomies would be wrong, it would not allow one to avoid skepticism,

<sup>1</sup> The *Critique of Pure Reason* is quoted with the abbreviation 'KrV' and the page number of the first edition ('A') and the second edition ('B'). The other writings by Kant are quoted with the abbreviation of the title, followed by the volume, eventually sub-volume, and page number of the Academy Edition. Quotations from the *Prolegomena*, the *Critique of the Power of Judgement* and the *Jäsche-Logik* also indicate the paragraph number. Quotations from the *Reflexionen* also indicate each *Reflexion's* number and the dating established by Adickes. Quotations from the 'transcripts' of Kant's lectures also indicate the dating of the lecture (not the dating of the transcript itself, which may have been many years later). For a list of the abbreviations and English translations used see below, 529–30. Every translation used reports the pagination of the German edition on the border of the page. So, the quotations do not report the page number of the translations.

<sup>2</sup> See *L. Blomberg* (beginning of the 1770s), xxiv.1:282. Kant denies that subcontrary judgments are opposite in a proper sense, because they can both be simultaneously true: see, e.g., *L. Pölitz* (beginning of the 1780s), xxiv.2:584; *Warschauer L.* (beginning of the 1780s), 633; *L. Dohna* (end of the 1780s or later), xxiv.2:770.

and it would fail indirectly to prove transcendental idealism and accept its 'Copernican revolution'.

Given the importance of this analysis, it is surprising that the *Critique of Pure Reason* and the *Prolegomena to Any Future Metaphysics*, published only two years after the *Critique*, contain very different accounts of the relation between theses and antitheses of the mathematical antinomies. According to the first *Critique*, the theses of the first two antinomies are composed of affirmative judgments of the form 'A is B'. The antitheses are composed of infinite judgments of the form 'A is non-B'. These judgments are contradictorily opposed only if a certain condition takes place. If this condition actually takes place, the theses and the antitheses are true and demonstrable, and the mathematical antinomies arise. Given transcendental idealism, this condition does not take place, the theses as well as the antitheses are false, and the antinomies do not arise.<sup>3</sup> According to the *Prolegomena*, at least at first sight, the theses are composed of affirmative judgments of the form 'A is B'. The antitheses are composed not of infinite, but of negative judgments of the form 'A is not B'. Given transcendental idealism, the concept of the subject common to both judgments is inconsistent. Therefore, by the rule '*non entis nulla sunt praedicata*', both judgments are false.<sup>4</sup>

These two accounts seem to ascribe two mutually irreducible logical forms to the mathematical antinomies, in particular to the antitheses. They allege two different reasons for the falsity of the theses and the antitheses, and they solve the antinomies with two different arguments. It is not at all evident if and how these two treatments can be seen as two different explanations of the same error, or if they can somehow be reconciled with each other. What is the logical form of the judgments opposed in the mathematical antinomies? Are they opposed by contradiction, as the *Prolegomena* suggest, or by another type of opposition, which the first *Critique* describes? Are the antitheses negative or infinite judgments? Do the *Critique* and the *Prolegomena* provide two distinct solutions of the mathematical antinomies, or do they provide two different versions of the same argument?

This enquiry purports to show that the conflict between the two accounts of the relationship of theses and antitheses in the *Critique of Pure Reason* and in the *Prolegomena* is only apparent. The idea of the existence of such a conflict depends on a misinterpretation of paragraphs 52b–52c of the *Prolegomena*. This interpretation is misled by the matching of a poorly chosen example and a very concise explanation of the relationship between the theses and the antitheses in the exposition of the *Prolegomena*. A different interpretation of the *Prolegomena* avoids the conflict with the first *Critique* and renders the two accounts of the mathematical antinomies consistent with each other.<sup>5</sup>

<sup>3</sup> See *KrV*, A 502–27/B 530–55.

<sup>4</sup> See *Proleg.*, iv:341–42.

<sup>5</sup> Ristitsch [1910], Llewelyn [1964], 173–4, and Lee [1989] already noted this problem. The first and the third author adhered to the interpretation of the *Prolegomena*

The discussion of this problem consists of five parts. The first part recalls the formulas of the opposed judgments and the argumentative structure of their proofs. The second part shows that the analysis of the relation between the theses and the antitheses plays a central role in the treatment of the mathematical antinomies in the first *Critique*. The third part expounds upon the analysis of the relation between the theses and the antitheses and the solution of the mathematical antinomies which can be found in the *Prolegomena*. The fourth part shows that, on the most natural reading of the *Prolegomena*, the analysis of the relationship between the theses and the antitheses in the *Critique* is incompatible with the one of the *Prolegomena*, and rejects an evolutive explanation of this contrast. The fifth part shows that the most natural reading of the *Prolegomena's* text rests on weak textual basis, it argues for another interpretation, and it shows that this interpretation renders the accounts of the *Critique* and of the *Prolegomena* consistent with each other. On the contrary of what *prima facie* seems, they do not outline two different solutions of the mathematical antinomies, but one and the same solution.

### 1. *Some Preliminary Information*

These statements oppose each other in the mathematical antinomies.<sup>6</sup>

#### *First antinomy:*

- |       |                                    |       |  |
|-------|------------------------------------|-------|--|
| [1T1] | The world has a beginning in time. | [1A1] | The world has existed eternally.         |
| [1T2] | The world has limits in space.     | [1A2] | The world does not have limits in space. |

#### *Second antinomy:*

- |       |  |       |   |
|-------|--|-------|---|
| [2T1] | Every composite substance in the world consists of simple parts. | [2A1] | No composite substance in the world consists of simple parts. |
| [2T2] | Simple substances exist in the world.                            | [2A2] | No simple substance exists in the world. <sup>7</sup>         |

According to Kant, every judgment comprised in the theses ascribes some sort of finite magnitude to the world, and every judgment comprised in

upon which I will expound in § 4 and criticize in § 5. Llewelyn proposed an evolutive explanation of the change in Kant's position. For the sake of brevity, I will not comment in detail on Llewelyn's interpretation, as Walsh [1976], 84–5 does. I only note that an evolutive account such as Llewelyn's is not plausible for the reasons mentioned below, 524. The conciliation of the *Critique* with the *Prolegomena* which is proposed below, 526–9 compared with Llewelyn's account, presents the advantage of rendering Kant's position coherent.

<sup>6</sup> See *KrV*, A 426–27/B 454–55, A 434–35/B 462–63, A 504–5/B 532–33; *Prol.*, § 51, iv:339; *Fort.*, xxiv:288–89.

<sup>7</sup> Kant affirms that the judgments which construct the theses and the antitheses of the first antinomy are opposed to each other. The whole formula of 2T2 and 2A2 is respectively: 'simple substances and composite substances exist in the world'; 'simple substances do not exist in the world, but only composite substances'. 2T1, 2T2, 2A1 and 2A2, as well as their proofs, share the presupposition that some composite substances

the antitheses ascribes an infinite magnitude to the world. The theses and the antitheses can be reformulated as judgments of the form 'x is finite' or 'x is infinite':

- [1T1] The world is finite with regard to time [gone by since its origin].  
 [1A1] The world is infinite with regard to time [gone by since its origin].  
 [1T2] The world is finite with regard to its extension.  
 [1A2] The world is infinite with regard to its extension.  
 [2T1] Every composite substance in the world is finite with regard to the decomposition of its parts.  
 [2A1] Every composite substance in the world is infinite with regard to the decomposition of its parts.<sup>8</sup>

Only 2T2 and 2A2 cannot be formulated in this way. When Kant introduces the solution of the mathematical antinomies in the seventh section of the antinomies chapter of the first *Critique*, it is not accidental that he leaves out 2T2 and 2A2.<sup>9</sup> They are a particular couple of judgments compared to the other judgments which are opposed in the mathematical antinomies.<sup>10</sup> For the sake of brevity, in the exposition of the solution of the mathematical antinomies I will not discuss the peculiarities due to the form of 2T2 and 2A2.

Human reason is naturally inclined to hold:

- 1) that 1T1 and 1A1, 1T2 and 1A2, 2T1 and 2A1, 2T2 and 2A2 are opposed to each other in the technical sense of this expression: they cannot both be true at the same time and from the same point of view;
- 2) that they are contradictorily opposed, i.e., that they cannot both be false at the same time and from the same point of view.<sup>11</sup> Two contradictory judgments are determinately one true and the other false.

exist (see *KrV*, A 434–36/B 462–64). The second part of 2T2 ('and composite substances') and the second part of 2A2 ('but only composite substances') are not part of the opposition. Thus, they are omitted from the formulas given above.

<sup>8</sup> See *KrV*, A 504–6/B 532–34. 2T1 and 2A1 mean: the division of composite substances in the world must stop after a finite series of steps, or it can go on endlessly. To be sure, in A 505/B 533 Kant formulates 2T1 and 2A1 differently, writing that "the quantity of parts in a given appearance is in itself neither finite nor infinite". This version of 2T1 and 2A1 implies that a phenomenal object cannot be constituted by an infinite quantity of simple parts. This seems to be false, and Kant does not argue for it. A 523–27/B 551–55 and the other formulations of 2T1 and 2A1, anyway, make clear that these sentences are not concerned with the question, if the quantity of the parts of every phenomenal object is finite or infinite, but with the question, if phenomenal objects have or do not have simple parts, no matter if in a finite or infinite quantity.

<sup>9</sup> See *ibid.*

<sup>10</sup> 2T2 is a mere corollary of 2T1. 2A2 is explained and proven through concepts and assumptions which are typical of transcendental idealism (see, e.g., Guyer [1987], 410; Kreimendahl [1998], 432; Malzkorn [1999], 186–7).

<sup>11</sup> See *KrV*, A 501/B 529. Man naturally adheres to transcendental realism. Kant holds that, if transcendental realism is true, the proofs of the opposed judgments are

The truth of the second supposition is necessary for the correctness of the proofs of the opposed judgments. The proofs of 1T1, 1A1, 1T2, 1A2, 2A1 and 2A2 are delineated in these steps:

- 1) they assume the judgment opposed to the *demonstrandum* as true;
- 2) they derive a contradiction from this assumption;
- 3) they infer the falsity of the judgment opposed to the *demonstrandum* from this contradiction, by *reductio ad absurdum*;
- 4) they infer the truth of the *demonstrandum* from the alternative that either the *demonstrandum* or its opposed judgment must be true and from the falsity of the latter.

The proof of 2T1 follows the same first two steps. Then it goes on as follows:

- 3) from the contradiction derived at step 2 it infers, by a *reductio ad absurdum*, that either the judgment opposite to the *demonstrandum* (2A1) or another judgment  $p$ , whose truth is presupposed by the argument at step 2, is false;
- 4) it proves that a false judgment follows from non- $p$ ;
- 5) it infers the falsity of 2A1 from the alternative stated at step 3 and the falsity of non- $p$ ;
- 6) it infers the truth of the *demonstrandum* from the alternative that either the *demonstrandum* or its opposed judgment must be true and from the falsity of the latter.<sup>12</sup>

The last step (respectively numbers 4 and 6) is correct only if the assumption refuted in the first part of the proof is the contradictory opposite of the *demonstrandum*.<sup>13</sup> Otherwise, it is not possible to infer the truth of the *demonstrandum* from the falsity of the refuted assumption, and the proofs are wrong.

## 2. *The Analysis and the Solution of the Mathematical Antinomies in the First Critique*

The analysis of the relationship between theses and antitheses plays a decisive role in the treatment of the mathematical antinomies in the first

valid (see Malzkorn [1999], 115–8). The proofs are correct only if the judgments are contradictorily opposed (Malzkorn does not agree with this). So, man is naturally inclined to hold that the judgments are opposed by contradiction.

<sup>12</sup> See *KrV*, A 426–43/B 454–71. For a formal exposition, see Malzkorn [1999], 121–90. The proof of 2T2 is direct, but this analysis leaves out the peculiarities of 2T2 and 2A2. The proposition  $p$ , which is a premise of the proof of 2T1, is: ‘it is possible to remove all composition in thought’. Kant outlines a more complicate proof for 2T1 than for the other propositions in order to make sure that it is 2A1, and not  $p$ , which is false.

<sup>13</sup> Kant proves 2T1 by refuting 2A1, which is formally opposed to 2T1 by contrariety, not by contradiction. The structure of the proof seems to exclude the possibility that some substances are divisible into simple parts and others are endlessly divisible. If it is thus, 2T1 and 2A1 are determinately one true and the other false, as actual contradictory judgments are.

*Critique*. The solution of the mathematical antinomies starts from the results of this analysis and presupposes its correctness. Moreover, the mathematical antinomies accomplish their anti-skeptical, demonstrative and pedagogical function only if the analysis of the relationship between the theses and the antitheses is correct.

### 2.1 *The Relationship between the Theses and the Antitheses*

The analysis of the relationship between the theses and the antitheses in the first *Critique* aims to leave open the possibility that the opposite judgments are not contradictorily opposed to each other. In the mathematical antinomies there is not a traditional contradictory opposition between affirmative and negative judgments, but an opposition *per disparata* between affirmative and infinite judgments. These judgments are determinately the one true and the other false only if a certain condition takes place, i.e., if the world has a determinate magnitude. Otherwise, they are both false.

Kant introduces the analysis of the relation between the theses and the antitheses with some examples. Zeno of Elea claimed that the world “is neither in motion nor at rest, and it is neither like nor unlike any other thing. To those who judged him, it appeared that he wanted entirely to deny two mutually contradictory propositions, which is absurd”. As the universe is in no place—rather, it comprises every place in itself—it is neither in motion from one place to another, nor at rest in the same place. Similarly, “if the world-whole (*Weltall*) includes in itself everything existing, then it is neither like nor unlike any *other thing*, because there is no *other thing* outside it, with which it might be compared”. “If someone said that a body either smells good or smells not good”, he would be forgetting another possibility, “namely that a body has no smell [...] at all, and thus both conflicting proposition can be false”.<sup>14</sup>

In each of these examples we can distinguish three mutually disjunctive and conjointly exhaustive possibilities. The first two are expressed by the opposed judgments. The third one is expressed by the negation of the condition necessary for their truth:

the world is at rest in the same place, it is in movement from a place to another one or it is not in any place;  
 a being *x*, external to the world and comparable with it, exists and the world is like *x*; a being *x*, external to the world and comparable with it, exists and the world is unlike *x*; no being which is external to the world and comparable with it exists;  
 the body *x* smells good, the body *x* smells not good (bad), the body *x* has no smell at all.

<sup>14</sup> A 502–3/B 530–31. Kant formulates the last example with a universal judgment. For the sake of simplicity, I have modified the translation of the example to express it with a singular judgment. Only oppositions between singular judgments are at stake in the mathematical antinomies. Considering oppositions between universal judgments as well would require a useless prolongation of the analysis.

The opposite judgments are determinately one true and the other false only if the third possibility does not take place. Otherwise, they are both false.

Kant distinguishes this type of opposition from the traditional contradictory opposition between two subject-predicate judgments, for instance 'the body  $x$  smells good' and 'the body  $x$  does not smell good'. In this case the two opposite judgments cover the whole range of possibilities. This is because the second judgment is the mere negation of the first one. It only asserts that the state of affairs which the first judgment describes does not take place. For instance, 'the body  $x$  does not smell good' is true if  $x$  smells bad as well as if  $x$  does not have any smell at all. Therefore, if one of these two judgments is false, the other is true. Instead, "in the previous opposition (*per disparata*)" between 'the body  $x$  smells good' and 'the body  $x$  smells not good', the second judgment was not a mere negation of the first. "The contingent condition of the concept of body (of smell) remained in the case of the conflicting judgment, and thus it was not ruled out by it; hence the latter judgment was not the contradictory opposite of the former".<sup>15</sup>

In the mathematical antinomies there is no traditional contradictory opposition, but an opposition *per disparata*. Kant explains this in detail with reference to the relationship between 1T2 and 1A2:

if I say that as regards space either the world is infinite or it is not infinite (*non est infinitus*), then if the first proposition is false, its contradictory opposite, "the world is not infinite", must be true. [...] But if it is said that the world is either infinite or finite (non-infinite), then both propositions could be false. For then I regard the world as determined in itself regarding its magnitude, since in the opposition I not only rule out its infinitude, and with it, the whole separate existence of the world, but I also add a determination to the world, as a thing existing in itself, which might likewise be false, if, namely, the world were *not given at all as a thing in itself*, and hence, as regards its magnitude, neither as infinite nor as finite.<sup>16</sup>

With regard to the magnitude of the world, one can distinguish three possibilities. The world is "determined in itself regarding its magnitude", i.e., it has a determinate magnitude, or it is indeterminate regarding its magnitude. If the world has a determinate magnitude, this magnitude is finite, that is, it can be expressed with a number, or it is infinite, that is, "greater than any number".<sup>17</sup> If the world is indeterminate regarding its magnitude, it has neither a finite nor an infinite magnitude. The judgments 1T2 ('the world is finite with regard to its extension') and 1A2 ('the world is infinite with regard to its extension') are determinately one true and

<sup>15</sup> *KrV*, A 503/B 532 (translation modified); see *M. Mrongovius* (1782–83), xxix. 1,2:809.

<sup>16</sup> *KrV*, A 504–5/B 532–33 (translation modified).

<sup>17</sup> *KrV*, A 432/B 460. According to Kant, this concept of actual infinity is legitimate only for those who accept transcendental realism. Transcendental idealism admits only potential infinite. On the relationship between these two concepts of infinite in the antinomies, see Falkenburg [2000], 164–72, 218–27, 249–54.



the other false only if the world has a determinate magnitude. Otherwise, they are both false. Thus, 1T2 and 1A2 are opposed *per disparata*, like 'the body  $x$  smells good' and 'the body  $x$  smells not good'.

How should 1T2 and 1A2 be formulated to build a traditional contradictory opposition, in which each judgment is the mere negation of the other? Instead of 1A2 ('the world is infinite as regards space' (*Die Welt ist dem Raume nach unendlich*)) should be 1A2\*: 'the world is not finite as regards space' (*Die Welt ist dem Raume nach nicht endlich*), or, in the place of 1T2 ('the world is finite as regards space' (*Die Welt ist dem Raume nach endlich*)) should be 1T2\*: 'the world is not infinite as regards space' (*Die Welt ist dem Raume nach nicht unendlich*). The difference between 1A2 and 1A2\* is subtle, but it is important not to confuse the opposition *per disparata*, which occurs in the antinomies, with the traditional contradictory opposition. Let us try to clarify it a little more.

1A2 and 1A2\* have different truth-conditions. If 1T2 is false, 1A2\* is certainly true and it can be inferred from the falsity of 1T2. 1A2 is true only if 1T2 is false and the world has a determinate magnitude.

1A2 and 1A2\* have different logical forms as well. 1A2\* is a negative judgment, because its copula is accompanied by a negative particle. 1A2 is an affirmative judgment, because its copula is not accompanied by a negative particle. In 1A2 the predicate 'unendlich' is composed of the negative prefix 'un-' and the adjective 'endlich'. The particle 'un-' does not refer to the copula 'ist', but to the adjective 'endlich'. It contributes to determine the meaning or 'content' of the predicate and of the judgment by expressing a negation or privation. Logic "abstracts from all content of cognition" and "concerns itself merely with the form of thinking [...] in general".<sup>18</sup> The fact that the predicate contains or does not contain a negation is not relevant to the logical form of a judgment. Kant calls 'infinite' an affirmative judgment whose predicate is accompanied by the negation. 1A2 is an infinite judgment, whereas 1A2\* is a negative judgment.

The linguistic or superficial form of 1A2 and that of 1A2\* are very similar. 'Nicht endlich' is synonymous of 'unendlich'. So, 1A2 can be paraphrased in this way: '*die Welt ist dem Raume nach nicht endlich*'. 1A2\* and the paraphrase of 1A2 are perfectly identical, even if the former is a negative judgment and the latter is an infinite one.

The similarity between the superficial form of 1A2 and 1A2\* makes it easy to mistake the opposition *per disparata* between 1T2 and 1A2 with a traditional contradictory opposition between an affirmative judgment and a negative one. If the world has a determinate magnitude, this error has no consequences. If the world does not have a determinate magnitude, this error may let us incorrectly infer 1T2 from the falsity of 1A2 and 1A2 from the falsity of 1T2. In this case the opposition between 1T2 and 1A2 is called 'dialectical opposition', because it deceptively seems, but it is not, a genuine opposition between contradictory judgments.

The distinction between negative and infinite judgments, the traditional contradictory opposition, the opposition *per disparata* and the

<sup>18</sup> *KrV*, A 131/B 170; see *KrV*, A 54/B 78.

dialectical opposition is essential for a correct analysis of the relationship between the theses and the antitheses of the mathematical antinomies. All judgments opposed in the mathematical antinomies can be formulated as affirmative judgments of the form 'x is finite' and infinite judgments of the form 'x is infinite'.<sup>19</sup> The relationship between the theses and the antitheses of the mathematical antinomies is identical to the relationship between 1T2 and 1A2, so it is easy to take it for a traditional contradictory opposition. If one does not distinguish negative and infinite judgments, the traditional contradictory opposition and the opposition *per disparata*, one cannot acknowledge that both theses and antitheses may be false. The solution of the mathematical antinomies proves that it is this possibility which actually takes place.

## 2.2 *The Solution of the Mathematical Antinomies*

The solution of the mathematical antinomies in the *Critique of Pure Reason* begins with the results of the analysis of the relationship between the theses and the antitheses and it presupposes its correctness. The analysis established that, if the world does not have a determinate magnitude, the theses and the antitheses are false and they are not contradictorily opposed to each other. The solution of the mathematical antinomies proves that the world does not have a determinate magnitude with a two-part argument:

- 1) the world has a determinate magnitude only if it is a thing in itself. Given transcendental realism, the world is a thing in itself and it has a determinate magnitude. So, it is finite or infinite as regards to its extension in space, its origin in time and to the quantity of the final elements of every substance which is comprised in it. The judgments opposed in the mathematical antinomies are determinately one true and the other false. We can infer the truth of each one from the falsity of the other. The proofs of the opposite judgments make use of this rule and are valid. Therefore, we should hold as true the judgments which construct the theses as well as those which construct the antitheses, even if they are contradictorily opposed to each other. The mathematical antinomies arise.
- 2) The *Transcendental Aesthetic* has proven the falsity of transcendental realism and the truth of transcendental idealism. Given transcendental idealism, the world is not a thing in itself and it does not have a determinate magnitude. From this it follows in the first place that the world is not determinately finite or infinite. We cannot infer the truth of each judgment which composes a thesis or an antithesis from the falsity of its opposite. The proofs of the theses and of the antitheses make use of this inference, so they are wrong. The theses and the antitheses are not proven. In the second place, from the fact that the world does not have a determinate magnitude it follows that

<sup>19</sup> See above, 509.

it is neither finite, nor infinite. The theses are false as well as the antitheses.<sup>20</sup>

Let us examine the first part of the argument. For an object to exist *in itself*, that is, independently of its being known or knowable, the complete series of all conditions that, from various points of view, the object presupposes has to subsist or to have occurred. With reference to time, the complete series of its previous states should have occurred. With reference to its extension, the totality of its parts should exist. With reference to each one of those parts, every part in which that part can in turn be divided should exist. In other terms, for things in themselves holds the principle that, “if the conditioned [*das Bedingte*] is given, then the whole series of all conditions for it is also given”.<sup>21</sup>

Transcendental realism holds that the objects of our senses exist in themselves, independently of our knowledge, and that this knowledge mirrors or correctly reproduces ‘things in themselves’ in our representations. For the transcendental realist, objects as we know them are identical with objects as they are in themselves.

As he identifies objects of our senses with things in themselves, the transcendental realist can formulate this ‘dialectical syllogism’:

if the conditioned is given (i.e., it exists and it can be known),  
then the whole series of all conditions for it is also given (*principle valid for things in themselves*)  
the conditioned is given (*in experience, by means of sensations;*  
*but transcendental realists identify objects of sensation and*  
*things in themselves*)

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the whole series of its condition, i.e., the world, is also given.<sup>22</sup>

It follows from the syllogism that, as the objects of the senses—for the transcendental realist—can be known as they are in themselves, the world as well exists and it can be known as it is in itself.

Completeness is a crucial characteristic of the idea of the world. The world is “the *absolute* totality of the sum total of existing things”, “the *absolute* unity of the series of conditions of appearance”, the “sum total of *all* appearances”.<sup>23</sup> This whole should not be understood as a class, of which appearances are elements in the sense of modern class theory, but as a mereological totality, of which appearances are parts, linked to each other by causal, spatial, succession and coexistence relations.<sup>24</sup> It is not a *totum*

<sup>20</sup> See *KrV*, A 504–5/B 532–33. The proofs of the opposite judgments are wrong even because they employ some premises which are true and some concepts which are admissible only if one accepts transcendental realism, but not if one accepts transcendental idealism. For the sake of brevity, the present analysis does not consider this point.

<sup>21</sup> *KrV*, A 497/B 525; see *KrV*, A 307–8/B 364.

<sup>22</sup> See *KrV*, A 497/B 525, A 307/B 364–65.

<sup>23</sup> See respectively *KrV*, A 419/B 447, A 334/B 391, A 506–7/B 534–35 (italics mine).

<sup>24</sup> See Falkenburg [2000], 197.

*analyticum*, that is, a whole whose parts exist and can be thought of only in relation to it. It is a *totum syntheticum*, that is, a whole which results from the composition or collection of parts that may exist and be thought even separately from it. It exists only if their collection is complete.<sup>25</sup>

If the world is given as it is in itself and if it is characterized by completeness, it is completely determined in its greatness. It is determinately finite or infinite with regard to the three types of 'greatness' which are at stake in the mathematical antinomies. It exists from a certain time (1T1) or it is infinite with regards to its origin (1A1). The quantity of its parts is finite (1T2) or infinite (1A2). The divisibility of each part stops at some simple indivisible component (2T1), whose existence has to be acknowledged (2T2), or it goes on infinitely (2A1) and no simple substance exists (2A2).

Given transcendental realism, then, the judgments which construct the theses and the antitheses are contradictorily opposed to each other. Their proofs are valid, they are proven and therefore true. So the mathematical antinomies arise.

The second part of the argument builds upon the results of the previous sections of the *Critique*.<sup>26</sup> The *Transcendental Aesthetic* has proven that transcendental idealism should be preferred to transcendental realism. Unlike transcendental realism, transcendental idealism distinguishes objects sharply, as they are in themselves from objects, as we know them. The latter "are nothing but appearances, i.e., mere representations, which, as they are represented [...], have outside our thoughts no existence grounded in itself".<sup>27</sup> They do not possess any property which cannot be object of experience. On the contrary, things in themselves are totally unknown to us.<sup>28</sup> Given this distinction between appearances and things in themselves, not every rule valid for ones is valid for the others. The major premise of the dialectical syllogism is true for things in themselves, not for appearances. The minor premise is true only for appearances, because we cannot know any thing in itself. To be true, the major premise has to take the subject ('the conditioned') as a thing in itself, the minor premise as an appearance. The premises take a term in two different meanings. Given transcendental idealism, the syllogism is not valid:<sup>29</sup> it does not show that the world is given as a thing in itself.

The solution of the mathematical antinomies in the *Critique of Pure Reason* rests on the fact that the world is not a thing in itself,<sup>30</sup> but, on the

<sup>25</sup> See Allison [1983], 43.

<sup>26</sup> Kant recalls the central theses of transcendental idealism right before introducing the solution of the mathematical antinomies (see *KrV*, A 490–97/B 518–25).

<sup>27</sup> *KrV*, A 490–91/B 518–19.

<sup>28</sup> See e.g. *KrV*, A 277–78/B 333–34.

<sup>29</sup> This error is called '*quaternio terminorum*' and '*sophisma figurae dictionis*' (see *KrV*, A 498–501/B 525–29).

<sup>30</sup> The thesis that the "sum-total of appearances" is a thing in itself would sound quite paradoxical in Kant's Critical philosophy. The world is a *totum syntheticum* of appearances: it exists only if the appearances which compose it exist. They exist only if

contrary, is strictly related to our experience. In the empirical regress in time, from an event to its conditions of possibility, and in space, from an object to the objects which are external to it, “there can be encountered *no experience of an absolute boundary*, and hence no experience of a condition as one that is *absolute unconditioned empirically*”.<sup>31</sup> “One can [...] concede that the decomposition” of the space “could never do away with all composition”.<sup>32</sup> On the other hand, “I cannot say the world is *infinite* in past time or in space”.<sup>33</sup> “It is by no means permitted to say of such a whole, which is divisible to infinity, that *it consists of infinitely many parts*”.<sup>34</sup> This is because we can never have experience of infinitely many objects or events which follow one another in space or in time, nor can we “exhibit any infinite multiplicity or the taking together of this multiplicity into one whole”.<sup>35</sup> If the world exists only in our experience and there cannot be any experience of a finite or (actually) infinite world, then the world is neither finite nor infinite.

So, the mathematical antinomies have a ‘negative solution’. As transcendental realism is false and transcendental idealism is true, the world is neither finite, nor infinite. It is indeterminate or indefinite with regards to its extension in space, its origin in time and to the quantity of the final elements of every object which is comprised in it. The proofs of the opposite judgments are wrong and the theses are false as well as the antitheses.<sup>36</sup> The anti-skeptical, demonstrative and pedagogical functions of the mathematical antinomies rely on this ‘negative solution’.

### 2.3 *The Anti-Skeptical, Demonstrative and Pedagogical Functions of the Solution of the Mathematical Antinomies*

Kant ascribes an anti-skeptical, a demonstrative and a pedagogical function to the solution of the mathematical antinomies.<sup>37</sup> The anti-skeptical function consists of its capacity to avoid reason’s fall into skepticism as a consequence of the “contrast of reason with itself”. The demonstrative

they are linked to some actual experience. Thus, the existence of the world depends on experience as well. Things in themselves are completely independent of experience.

<sup>31</sup> *KrV*, A 517/B 545.

<sup>32</sup> *KrV*, A 525/B 553.

<sup>33</sup> *KrV*, A 520/B 548.

<sup>34</sup> *KrV*, A 524/B 552.

<sup>35</sup> *Ibid.*

<sup>36</sup> Besides this ‘negative solution’, Kant gives an ‘affirmative solution’ of the mathematical antinomies. This is the explanation of “*how the empirical regress is to be instituted* so as to attain to the complete concept of the object” (*KrV*, A 510/B 538; see A 515–16/B 543–44). No experience can give us that concept. It is the ideal to which we have to tend in enlarging and systematizing our empirical knowledge.

<sup>37</sup> Kant ascribes these functions to all antinomies, employing ‘antinomy of pure reason’ as a collective term. Some interpreters (e.g., Guyer [1987], 406–7) deny that the dynamical antinomies provide an indirect proof of transcendental idealism. If this is true, only the mathematical antinomies have a demonstrative and a pedagogical function. However things may be, this paragraph considers only the mathematical antinomies. It also leaves out the systematic role of the antinomies, because only the treatment of

function consists of the possibility of reformulating the solution of the antinomies as an indirect proof of transcendental idealism. The pedagogical function consists of the capacity of this indirect proof to persuade man to abandon transcendental realism and adhere to the Critical point of view. Let us see how the solution of the first *Critique* carries out these functions.

The antinomies threaten to lead everyone who raises the problems of rational cosmology to skepticism. Until one begins a critical inquiry of the extension, foundation and limits of human knowledge, one naturally adheres to transcendental realism. On the basis of this theory, together with assumptions and definitions which Kant considers obvious, the antinomies arise “naturally” and “unavoidably” in the human mind.<sup>38</sup> Transcendental realists can prove couples of contradictorily opposed judgments with apparently valid arguments from premises which seem true. As it is well-known, from a single contradiction it is possible to prove every judgment and its negation with correct inferences. Once admitted, as Kant does, that consistence is a necessary condition for truth, the existence of a real “contradiction of reason with itself” would sanction the incapacity of human reason to attain the truth. The antinomies are a privileged way to skepticism.

In the face of this danger, man cannot avoid taking sides. His interest in science, morals and religion induces him to look with favor sometimes to the theses, sometimes to the antitheses. It does not allow him to lose interest in the antinomial conflict, as if it were not an important problem.<sup>39</sup> It only remains for him to embrace skepticism, or to prove that the theses and the antitheses are not contradictorily opposed. The solution of the antinomies allows man to choose the second alternative, to rescue the power of reason to attain the truth and to avoid falling into skepticism.<sup>40</sup>

The solution of the antinomies refuses skepticism only by assuming the validity of transcendental idealism among its premises. It is not an easy assumption to make, because it implies the acceptance of the long and complex arguments of the *Transcendental Aesthetic* and the *Transcendental Analytic*. By reformulating the solution of the antinomies as an indirect proof of transcendental idealism, Kant reverses the relation of dependence between it and the solution of the antinomies. Transcendental idealism is no longer a premise necessary for solving the antinomies. On

the dynamical antinomies of pure reason, not that of the mathematical ones, is a model for the treatment of the antinomies of moral philosophy, aesthetics and teleological consideration of living beings.

<sup>38</sup> Kant describes this process in detail in *KrV*, A 408–21/B 435–49. On the naturalness and unavoidability of the antinomies see e.g. *KrV*, A 407/B 433–34, A 441–42/B 449–50, A 462/B 490.

<sup>39</sup> See *KrV*, A 462–76/B 490–504.

<sup>40</sup> In the Critical period, Kant was very hostile to this doctrine (see e.g. *KrV*, A 758/B 786 ff.) Many scholars hold the confutation of Humean skepticism to be one of the main goals of the *Critique of Pure Reason*. If so, the importance of the anti-skeptical function of the antinomies can hardly be exaggerated.

the contrary, the solution of the antinomies offers a new proof of transcendental idealism, independent of the *Transcendental Aesthetic* and the *Transcendental Analytic*.

The indirect proof of transcendental idealism is as follows. Transcendental realism holds that “the world is a whole existing in itself”. Transcendental idealism holds that “appearances in general are nothing outside our representations”. Transcendental realism and transcendental idealism are two mutually exclusive and jointly exhaustive theories of the relationship between knowing subjects and known objects.<sup>41</sup> Given transcendental realism, the world is either finite or infinite, but it is possible to refute both these alternatives, “according to the proof offered above for the antithesis on the one side and the thesis on the other”. Thus, transcendental realism is false and transcendental idealism is true.<sup>42</sup>

The pedagogical role of this indirect proof of transcendental idealism is even more important than its demonstrative function. From a demonstrative point of view, the indirect proof is useful, but it is not necessary. The *Transcendental Aesthetic* and the *Transcendental Analytic* constituted a long, direct proof of transcendental idealism, independent of the one provided by the antinomies.<sup>43</sup> The latter proof is indispensable in persuading man to accept transcendental idealism only from a pedagogical point of view. If transcendental realism is true, man can hope to satisfy his inborn desire to know the world, the soul and God. Transcendental idealism denies the possibility of knowing the supersensible. The direct proof of the first part of the *Critique*, even if valid, is not persuasive enough to induce man to accept “a principle [transcendental idealism] that so narrows the field of its speculation” and “sacrifices in which so many otherwise shining hopes must entirely disappear”.<sup>44</sup> The indirect proof derived from the antinomies is more persuasive, because “a contradiction always carries with it more clarity of representation than the best connection [between the premises and the conclusion, exhibited by direct proofs], and thereby more closely approaches the intuitiveness of a demonstration”.<sup>45</sup> Only by reflecting on the “conflict of reason with itself” man can resign himself to abandon transcendental realism and accept the Copernican revolution of Critical philosophy.<sup>46</sup>

<sup>41</sup> See Allison [1983], 14 ff. For a critic, see Malzkorn [1999], 103, n. 45 and 112–3.

<sup>42</sup> *KrV*, A 506–7/B 535–36. Sometimes Kant describes this proof as an ‘experiment of pure reason’, similar to those which are carried out in natural science (see *KrV*, B xx–xxi, text and note; *Fort.*, xx:290–91).

<sup>43</sup> From a demonstrative point of view, direct proofs should be preferred to indirect proofs (see *KrV*, A 789/B 817). It follows that the argument exposed in the first part of the *Critique* is to be preferred to the argument drawn from the antinomies.

<sup>44</sup> *KU*, § 57, Anm. II, v:344.

<sup>45</sup> *KrV*, A 790/B 818.

<sup>46</sup> Kant emphasizes the pedagogical importance of the discovery of the antinomies no less than the importance of their solution: see e.g. *KpV*, v:107–8; *KU*, § 57, Anm. II, v:344–45.

The anti-skeptical, demonstrative and pedagogical functions of the antinomies allow human reason to go through a necessary path from the deceptive appearance of transcendental realism, through the reflection on the cosmological problems, the awakening from the “dogmatic slumber”, the experience of the “conflict of reason with itself” and the threat of skepticism, to the acceptance of transcendental idealism.<sup>47</sup>

Obviously, though, the solution of the mathematical antinomies can carry out these functions only if it is correct. Its correctness in turn depends on the correctness of the analysis of the relationship between the opposite judgments. If the theses and the antitheses were not composed of affirmative and infinite judgments, which are contradictorily opposed only on a certain condition, but affirmative and negative judgments, it would not be possible to deny both of them and assign the truth to a third alternative.<sup>48</sup> It would not be possible to avoid skepticism, to prove indirectly transcendental idealism and to transform this proof into a pedagogical instrument for reason reluctant to abandon realism. Thus, the analysis of the relationship between the theses and the antitheses is indispensable not only for the solution of the mathematical antinomies, but for their anti-skeptical, demonstrative and pedagogical functions as well.

### 3. *The Solution of the Mathematical Antinomies in the Prolegomena*

The treatment of the mathematical antinomies in the *Prolegomena* is much shorter than the treatment of the *Critique of Pure Reason*. After having introduced the theses and antitheses roughly in the same terms of the *Critique*, Kant explains the relationship between them by starting with an example. Then he explains why the theses are false as well as the antitheses: “of two mutually contradictory propositions both cannot be false *except if the concept underlying them both is itself contradictory*; e.g., the two propositions: a rectangular circle is round, and: a rectangular circle is not round, are both false”.<sup>49</sup> In this case a rule holds true which Kant recalls in the *Doctrine of Method* of the first *Critique*: “*non entis nulla sunt praedicata*, i.e., both what one asserts affirmatively as well as what one asserts negatively of the object are incorrect”.<sup>50</sup> “Underlying the first two antinomies [...] is a contradictory concept of this type”.<sup>51</sup> The subject of the opposite judgments is the inconsistent concept of “a sensible world existing for itself”. Thus, the theses are false as well as the antitheses.

Why should one understand the term ‘world’ in the theses and in the antitheses as ‘a sensible world existing in itself’? Why is this concept inconsistent?

<sup>47</sup> See *KrV*, A 761/B 789.

<sup>48</sup> On the importance of the infinite judgment in the antinomies, see Ishikawa [1990].

<sup>49</sup> *Prol.*, § 52b, iv:341 (italics mine).

<sup>50</sup> *KrV*, A 793/B 821.

<sup>51</sup> *Prol.*, § 52c, iv:341.



According to transcendental idealism, objects in space and time are not things in themselves, but appearances. "I must not say of that which I think in space or time: that it is in itself in space and time, independent of this thought of mine".<sup>52</sup> If what I think in space and time existed in itself as I represent it to myself, independently of its being represented, space and time would exist independently of my representations as well. According to transcendental idealism, "space and time [...] are nothing existing outside my representations, but are themselves only modes of representation". "It is patently contradictory to say of a mere mode of representation that it also exists outside our representation".<sup>53</sup> As a consequence, it is false that objects in space and time exist in themselves.

Once this premise has been established, the argument goes as follows:

- 1) If the world is finite or infinite, it is a thing in itself. "It is not possible to have experience of either an *infinite* space or infinitely flowing time, or of a *bounding* of the world by an empty space or by an earlier, empty time".<sup>54</sup> So, the world cannot be finite or infinite as an object of experience. It must be finite or infinite *in itself*, that is, independently of the possibility of having experience of its finiteness or infiniteness. In this case, the sensible world is a thing in itself.
- 2) "But this contradicts the concept of a sensible world, which is simply a sum total of the appearances whose existence and connection takes place only in representation, namely in experience, since the world is not a thing in itself, but is itself nothing but a mode of representation (*Vorstellungsart*)".<sup>55</sup> If the world had a finite or infinite magnitude, it would be "a sensible world existing for itself", an object of experience existing before and independently of any experience. This concept of 'world' is inconsistent.
- 3) The opposite judgments ascribe finiteness or infiniteness to the world. So, they adopt this inconsistent concept of 'world'. As *non entis nulla sunt praedicata*, they are false.

It is possible to apply this argument to the division of appearances in parts as well.<sup>56</sup> In this way one can prove that the theses as well as the antitheses of both mathematical antinomies are false. "The falsity of the presupposition" common to the theses and to the antitheses, concludes Kant, "consisted in the following: that something self-contradictory (namely, appearance as thing in itself) would be represented as being unifiable in a concept".<sup>57</sup>

The *Prolegomena* also expound upon various ideas which Kant had already explained in the discussion of the mathematical antinomies in

<sup>52</sup> Ibid.

<sup>53</sup> *Prol.*, § 52c, iv:341–42.

<sup>54</sup> *Prol.*, § 52c, iv:342.

<sup>55</sup> Ibid.

<sup>56</sup> See *Prol.*, § 52c, iv:342.

<sup>57</sup> *Prol.*, § 53, iv:343.

the first *Critique*. It is transcendental realism, with its identification of appearances and things in themselves, which leads man to ascribe finiteness or infiniteness to the sensible world, even if these may only belong to the world in itself.<sup>58</sup> Once transcendental realism has been accepted, the antinomies arise naturally and unavoidably.<sup>59</sup> In every antinomy “both thesis and antithesis can be established through equally evident, clear, and incontestable proofs”.<sup>60</sup> The “conflict of reason with itself” makes the skeptic rejoice<sup>61</sup> and it obliges human reason, as well as the attentive reader of the *Prolegomena*, to awake from the dogmatic slumber and undertake a critical examination of pure reason.<sup>62</sup> The *Prolegomena* emphasize this pedagogical function of the discovery of the antinomies more than the pedagogical function of their solution, that is, persuading man to endorse transcendental idealism. Nevertheless, there is no doubt that Kant considers the antinomies a privileged way to Critical philosophy in the *Prolegomena* as well.<sup>63</sup>

Other parts of the treatment of the mathematical antinomies in the *Critique* are absent in the *Prolegomena*. Here Kant expounds upon neither the proofs, nor the confutations of the opposed judgments, and he does not comment in detail on the solution of each mathematical antinomy. The solution of the antinomies in the *Prolegomena* makes clear why the theses and the antitheses are false, but it does not explain why their proofs are wrong. The *Prolegomena* do not report the indirect proof of transcendental idealism, which the first *Critique* draws from the solution of the antinomies. The introductory remarks to which Kant devoted no less than three paragraphs in the *Critique* are also absent in the work of 1783.<sup>64</sup> What is of greater relevance, however, is that which seems to be a radical divergence in the analysis of the relation between the theses and the antitheses.

#### 4. A Divergence between the First Critique and the Prolegomena?

According to the most natural reading of the *Prolegomena*, their analysis of the relationship between the theses and the antitheses is incompatible with the one of the *Critique of Pure Reason*. The *Prolegomena* seem to maintain that the judgments which construct the theses and the antitheses are opposed to each other like ‘a rectangular circle is round’ and ‘a rectangular circle is not round’, that is, as an affirmative and a negative judgment with the same subject, the same predicate and an affirmative

<sup>58</sup> See *Prol.*, § 52a, iv:339–40.

<sup>59</sup> See *Prol.*, § 51, iv:339.

<sup>60</sup> See *Prol.*, § 52a, iv:340; *Prol.*, § 52b, n., iv:341.

<sup>61</sup> See *Prol.*, § 52a, iv:340.

<sup>62</sup> See *Prol.*, § 50, iv:338; *Prol.*, § 52b, n., iv:341; *Prol.*, § 54, iv:347–48.

<sup>63</sup> See *Prol.*, iv:379–80.

<sup>64</sup> *KrV*, A 462–90/B 490–518.

or negative copula ('A is B'—'A is not B'). The theses and the antitheses have the typical form of contradictorily opposed judgments, but the inconsistency of the concept of their subjects makes both judgments false and allows no real contradictory opposition to hold.<sup>65</sup> The *Critique* maintains that the judgments which construct the theses and the antitheses are opposed as 'the body  $x$  smells good' and 'the body  $x$  smells not good'. They are not affirmative and negative judgments with the same subject, the same predicate and an affirmative or negative copula, but an affirmative and an infinite judgment with the same subject, an affirmative copula and different predicates, the first affirmative, the second negative ('A is B'—'A is non-B'). As a consequence, the opposition between the judgments which construct the theses and those which construct the antitheses is not a traditional contradictory opposition, but an opposition *per disparata*.

This is not a merely formal divergence. In the first *Critique* the correctness of the solution of the mathematical antinomies and of the indirect proof of transcendental idealism, as well as the anti-skeptical, demonstrative and pedagogical functions of this solution depend on the holding of an opposition *per disparata* between affirmative and infinite judgments. The *Prolegomena* seem to set forth a different analysis of their relationship. They abandon, at least apparently, the solution of the first *Critique*. They appeal to a different logical law and they present a new argument to establish the falsity of the theses and the antitheses.

Do both treatments of the mathematical antinomies really expound upon two different proofs, or is it possible to interpret them as two different versions of the same proof and to reconcile the differences between the *Critique* and the *Prolegomena*? In the first case, these questions have to be answered: which analysis of the opposite judgments is correct? Once the presuppositions of Kant's arguments are accepted as true, are both proofs valid? Can both proofs fulfill an anti-skeptical, demonstrative and pedagogical function? Why do the *Prolegomena* abandon the account of the first *Critique*? In the second case, an interpretation of the text of the *Prolegomena* which allows its reconciliation with the account of the first *Critique* should be given.

The hypothesis of evolution in Kant's position on the structure and solution of the mathematical antinomies is not convincing.<sup>66</sup> Kant published

<sup>65</sup> In the *Prolegomena*, Kant calls 'a rectangular circle is round' and 'a rectangular circle is not round' "two contradictory propositions", at the basis of which a contradictory concept lies (*Prol.*, § 52b, iv:341). If these judgments are both false, and if contradictory judgments are determinately one true and the other false, they are not contradictory judgments in strict sense. As the judgments opposed *per disparata*, the judgments in the example "collapse, because the condition collapses under which alone either of them would be valid" (*KrV*, A 503/B 531; italics mine). The condition at stake is the internal consistency of the subject concept.

<sup>66</sup> An evolutive explanation has been proposed, with more plausibility, for another contradiction in the discussion of the antinomies in the first *Critique*: that between the solution of the dynamical antinomies announced in A 505–6/B 533–34 and the solution expounded upon in A 532–65/B 560–93 (see Adickes [1889], 426, n. 1; Kemp Smith [1918], 506, 511–2).

the *Prolegomena* in 1783, two years after the *Critique of Pure Reason*. The *Prolegomena* and the writings which were not destined to publication—letters, *Reflexionen*, notes to the first edition of the *Critique*—do not offer any proof that, at that time, Kant became dissatisfied with the treatment of the mathematical antinomies of 1781, and that he looked for another one. On the contrary, the chapter on the antinomies in the 1787 edition of the *Critique* does not present any relevant variation in regard to the first edition.<sup>67</sup> Kant did not take pains to modify it to render it compatible with the treatment of the *Prolegomena*. In the second edition he published other unchanged pages which were no more compatible with its position of 1787, as those devoted to moral philosophy in the *Canon of Pure Reason*, which the *Groundwork* of 1785 and the *Critique of Practical Reason* of 1788 rendered out-of-date. Could he have changed his mind with regard to the solution of the mathematical antinomies as well, without updating the text of the first *Critique*? This does not seem to be the case. Since Kant devoted autonomous works to moral philosophy from 1785 on, the *Canon of Pure Reason* lost most of its importance.<sup>68</sup> This explains why Kant did not care to revise it in the second edition of the *Critique of Pure Reason*. Differently from the *Canon*, the antinomies still had a fundamental role in the Critical philosophy in 1787, as well as the transcendental deduction and the paralogisms.<sup>69</sup> As Kant, unsatisfied with the first redaction of these two chapters, rewrote them entirely for the 1787 edition, he would have rewritten the mathematical antinomies as well, if had no longer accepted the solution of 1781.

Upon exclusion of an 'evolutive' explanation of Kant's position, one can only declare the incompatibility of Kant's 1781 and 1783 texts,<sup>70</sup> or look for an interpretation of the *Prolegomena* which removes the contradiction to the first *Critique*. In the following paragraph, I will sketch the second alternative.

### 5. A Different Interpretation of the Treatment of the Mathematical Antinomies in the *Prolegomena*

Contrary to the first glance, the interpretation of the relationship between the theses and the antitheses in the *Prolegomena* which has been exposed rests on weak textual basis. No sentence in the *Prolegomena* explicitly states that the antitheses of the mathematical antinomies are negative judgments and are opposed to the theses in such a way that, had their subject not been inconsistent, there would be an authentic contradictory

<sup>67</sup> One can easily notice the differences between A and B, e.g., in R. Schmidt's edition of the *Critique of Pure Reason*.

<sup>68</sup> On the relationship between the *Canon of Pure Reason* and the *Critique of Practical Reason* see Landucci [1997], v–xii.

<sup>69</sup> Kant recalls the importance of the antinomies of 1781, together with those of the practical reason and of the aesthetic power of judgment, in *KpV*, v:107–8 and in *KU*, § 57, Anm. II, v:243.

<sup>70</sup> Ristitsch [1910], 493 does so.

opposition. With reference to the mathematical antinomies, the text reads: “now underlying the first two antinomies, which I call mathematical [...], is a contradictory concept of this type; and by this means I explain how it happens that thesis and antithesis are both false together”. “If I ask about the magnitude of the world with respect to space and time, for all of my concepts it is equally impossible to assert that it is finite as that it is infinite. [...] since the concept of a sensible world existing for itself is, in itself, contradictory, any solution to this problem as to its magnitude will always be false, whether one attempts to solve it affirmatively or negatively”.<sup>71</sup> The only expression which may refer to a contradictory opposition is the contraposition of an ‘affirmative attempt’ and a ‘negative attempt’ to solve the antinomies in the last sentence, but these are not technical expressions. ‘Negative attempt’ may refer to a negative judgment as well as to an infinite judgment, because both contain a ‘negative’ element as a part of the copula or of the predicate.

Which element of the text gives the reader the impression that Kant in the *Prolegomena* considers the opposite judgments an affirmative and a negative judgment of the form ‘A is B’ and ‘A is not B’? This depends on the fact that the explication of the relationship between the theses and the antitheses immediately follows the example of the opposition between ‘a circle is round’ and ‘a circle is not round’. The judgments in the example are really an affirmative and a negative judgment with the same subject, the same predicate and an affirmative or negative copula. Kant himself stresses that they are “two mutually *contradictory* propositions”.<sup>72</sup> In the last sentence before the explanation of the relationship between the theses and the antitheses he explains, with reference to the example: “the logical sign for the impossibility of a concept consists, then, in this: that under the presupposition of this concept, two *contradictory* propositions would be false simultaneously; and so, since in between these two a third proposition cannot be thought, through this concept *nothing at all* is thought”. Only in the case of contradictory opposites, not in that of affirmative and infinite judgments which are opposed *per disparata*, there is not any third possibility beside those described by the two judgments. So, if their subject were not inconsistent, the judgments in the example would actually be contradictorily opposed judgments.<sup>73</sup> Then Kant writes that “underlying the first two antinomies [...] is a contradictory concept of this type”.

This move from the example to the explanation of the relation between the theses and the antitheses suggests the reader to apply the type of opposition holding between the judgments in the example to the judgments opposed in the mathematical antinomies. An affirmative and a negative

<sup>71</sup> *Prol.*, § 52c, iv:342.

<sup>72</sup> For this and the following quotations, see *Prol.*, §§ 52b–52c, iv:341 (my italics).

<sup>73</sup> This prevents us from understanding ‘a rectangular circle is not round’ [*ein viereckichter Cirkel ist nicht rund*] as an infinite rather than a negative judgment, seeing in the example an opposition *per disparata*, and solving the contrast between the *Critique* and the *Prolegomena* in the simplest way.

judgment, like 'a rectangular circle is round' and 'a rectangular circle is not round', would oppose to each other in the mathematical antinomies. In the continuation of the text Kant explains why the concept of the subject common to the theses and the antitheses is internally inconsistent. As the judgments in the example are false because their subject is an inconsistent concept, so are the theses and the antitheses. The example seems to offer a good clue for understanding the structure of the mathematical antinomies.

Actually, the text of the *Prolegomena* only suggests this transposition of the logical form of the example to the relation between the theses and the antitheses, but it does not contain any unquestionable confirmation that the theses and the antitheses are really opposed as the judgments of the example. Kant only affirms that "underlying the first two antinomies [...] is a contradictory concept of this type". Like the judgments in the example, the theses and the antitheses are both false because the concept of their subject is inconsistent. Nevertheless, the rule '*non entis nulla sunt praedicata*' does not apply only to couples of opposed or contradictory judgments, but to every single judgment, be it affirmative, negative, or infinite. The theses and the antitheses are both false, even if the antitheses are not negative judgments and if they do not have the same predicate as the theses. It is enough that the antitheses have an internally inconsistent concept as subject, be they affirmative, negative, or infinite. Kant's statements are totally valid even if the antitheses do not have the form 'A is not B'.

This allows us to propose a different interpretation. In the *Prolegomena*, Kant does not decide what relation holds between the opposite judgments. He does not deny, nor does he state, that the theses and the antitheses are composed by affirmative and infinite judgments which are opposed *per disparata* and dialectically. He only explains why the theses are false as well as the antitheses, and he does this with an argument which is valid whatever form and relation to the theses the antitheses may have. The text of the *Prolegomena* is compatible with the explanation of the relationship between the theses and the antitheses of the first *Critique*.

A passage of the *Doctrine of Method* of the *Critique of Pure Reason*, some notices for the unfinished work on the *Progress of Metaphysics* and some *Reflexionen* confirm the compatibility of the *Critique* and the *Prolegomena*. In the *Doctrine of Method* Kant warns against employing indirect proofs in the sciences where one can substitute "that which is subjective in our representations for that which is objective, namely the cognition of what is in the object".<sup>74</sup> Indirect proofs rely on the contradictory opposition between two propositions, but in those sciences it may happen that "both propositions contradict each other only under a subjective condition that is falsely held to be objective, and that since the condition is false, both of them can be false, without it being possible to infer the truth of one from the falsehood of the other".<sup>75</sup> In metaphysics, in particular, sometimes

<sup>74</sup> *KrV*, A 791/B 819.

<sup>75</sup> *KrV*, A 792/B 820.

both, the affirmative as well as the negative part [...], have as their ground an impossible concept of the object, and then the rule holds that *non entis nulla sunt praedicata* [...], and one cannot arrive at cognition of the truth apagogically through the refutation of its opposite. So, for example, if it is presupposed that the sensible world is given in its totality *in itself*, then it is false that it must be *either* infinite in space or finite and bounded [...]. For appearances (as mere representations), which would yet be given *in themselves* (as objects) are something impossible, and the infinity of this imagined whole would, to be sure, be unconditioned, but would nevertheless (since everything in appearances is conditioned) contradict the unconditioned determination of magnitude that is presupposed in the concept.<sup>76</sup>

The first citations state that the opposition between the theses and the antitheses is a dialectical and not a contradictory one, as the *Critique* highlights. The last one explains that the concept of the subject of the theses and antitheses is inconsistent, as the *Prolegomena* emphasize. Kant places the two accounts in a single text, so he takes them to complement each other.

In the notices for the unfinished work on the *Progress of Metaphysics* we can find a similar hint for such an explanation of the apparent conflict. Here Kant maintains, as in the *Critique* but with different words, that in the mathematical antinomies there is not a “logical” or “analytical” contradictory opposition, but a “transcendental conflict of the synthetic opposition” between “two judgments which oppose each other as contraries”, every one of which “says more than what is required for the logical opposition”.<sup>77</sup> He also explains, as in the *Prolegomena*, that those who hold that the world is finite or infinite think of it as being a noumenal “absolute whole”, but this thought is self-contradictory.<sup>78</sup> The first statement allows him to show that the proofs are wrong. The second one explains the falsity of the theses and the antitheses.

Among the *Reflexionen*, the one published by Adickes as number 5962 and dated ‘about 1788–91’ suggests the same explanation in a more explicit way:

These two propositions [‘the world is infinite with regard to the space and time’ and ‘it is not (given as) infinite’] can both be false, because each one contains more than what is required for the contradiction. This is the logical solution of the antinomies. But they are also both false, because they contain an impossible condition, i.e., that the world is entirely given in space and in time (as well as [that] a *compositum* is entirely given [reference to the second antinomy]), and nevertheless that it is given in space and time. In fact the first sentence is grounded on the presupposition that a whole of the appearances is given in itself outside of the representations, which is contradictory. And this is the transcendental solution of the antinomy.<sup>79</sup>

<sup>76</sup> *KrV*, A 792–93/B 820–21.

<sup>77</sup> *Fort.*, xx:291.

<sup>78</sup> *Fort.*, xx:328.

<sup>79</sup> *Refl.* 5962 (about 1788–91), xviii:401–5, here 404.

If this interpretation is correct, the solutions of the mathematical antinomies of the first *Critique* and the *Prolegomena* are compatible with each other. The first *Critique* analyzes the relationship between the theses and the antitheses and relies on this analysis to show that their proofs are wrong. The *Prolegomena* leave completely off this proof and justify with a new argument the falsity of the theses and the antitheses.

The *Prolegomena* do not clarify which relationship holds between the theses and the antitheses, even if this is a fundamental part of the treatment of the antinomies in the *Critique of Pure Reason*, because their size and aims are much more modest. The *Critique* presents the mathematical antinomies in all details, it describes how man comes “naturally” and “unavoidably”<sup>80</sup> to hold the theses as well as the antitheses to be true, it exposes and comments upon their proofs. In order to solve the antinomies, it must show that the theses and the antitheses are not both true or that they are not contradictorily opposed to each other, and that the proofs of the false theses or antitheses are not valid. Showing that the theses and the antitheses are not contradictorily opposed is necessary not to show that they are both false, but to show that their proofs are not valid.<sup>81</sup> As they are only a compendium, the *Prolegomena* present a brief treatment of the mathematical antinomies. They explain neither the proofs of the theses and of the antitheses, nor the reasons why they are wrong. They only prove that the theses are false as well as the antitheses. To prove this, it is enough to show that their subject is inconsistent. It is not necessary to establish what kind of logical relation holds between them. Thus, there is nothing strange in the fact that the *Prolegomena*, differing from the *Critique*, do not explain what relation holds between the theses and the antitheses.

In the first *Critique* and in the *Prolegomena* Kant justifies the falsity of the opposite judgments with two different, but similar and compatible arguments. These two assumptions play a central role in both proofs: the world exists only in our experience, it is a ‘mode of representation’ and not a thing in itself; it is not possible to have experience of the finiteness or infiniteness of the world. The first assumption is typical of transcendental idealism. It is not accepted by transcendental realists.<sup>82</sup> Thus, both proofs presuppose the truth of transcendental idealism, which can be proven by the indirect proof expounded upon in the *Critique*. Both proofs rely on the impossibility of ascribing properties which cannot be object of any

<sup>80</sup> See above, 518, n. 38.

<sup>81</sup> In the first *Critique* Kant proves that the theses and the antitheses are false with the following argument: the world exists only in our experience; there cannot be any experience of a finite or (actually) infinite world; therefore, the world is neither finite, nor infinite. To set forth this argument it is not necessary to undertake a detailed analysis of the form of the theses and the antitheses. It is enough to prove that one cannot have experience of a finite or an infinite world.

<sup>82</sup> According to the *Critique* as well as to the *Prolegomena*, the negation of this assumption, that is, the admission that the sensible world is a thing in itself, is the wrong presupposition which determines the rise of the mathematical antinomies.



experience to a 'mode of representation'. The *Critique* makes use of this impossibility to hold that a finite or infinite world is not object of possible experience, that is, it is not 'really possible'. The *Prolegomena* make use of it to hold that the concept of a finite or infinite world is not only without 'real possibility', but it is without logical possibility as well, because it is internally inconsistent.<sup>83</sup> In fact, the *Prolegomena* follow the rule '*non entis nulla sunt praedicata*', which holds only for the internally inconsistent *nihil negativum*,<sup>84</sup> whereas the *Critique* does not mention this rule. Both texts conclude that the opposite judgments are false.

If we accept this interpretation of the solution of the antinomies in the *Prolegomena*, we must acknowledge that the positioning of the rectangular circle example right before the relationship between the theses and the antitheses is misleading. It gives the wrong impression that the *Prolegomena* ascribe to the antinomies a different relation from the one of the first *Critique*. The example of the rectangular circle may easily deceive the reader. If Kant had chosen an affirmative and an infinite judgment, rather than an affirmative and a negative one, the text would have been clearer. The example of the rectangular circle is not very appropriate even for those who maintain that the relation between the theses and the antitheses in the *Prolegomena* is actually that of contradiction. The predicate 'round' is analytically entailed in the concept 'circle', which is a part of the subject's concept, and it is contradictory to another concept which composes the subject, 'non-round', entailed by 'rectangular'. In the mathematical antinomies the predicates 'finite' and 'infinite' are neither comprised, nor excluded in the subject concept 'world'. The alternative to considering Kant's example and its positioning in the text inappropriate is holding that the *Prolegomena* contradict the *Critique*. Ascribing to Kant's text the first limit is surely the lesser of the two evils.

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<sup>83</sup> On the difference between logical and real possibility see *KrV*, A 596/B 624, n.

<sup>84</sup> On this point, I diverge from the interpretations of Brittan [1974] and Loparic [2000], 114–5.

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