Conjunction and Contradiction

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The Law of Non-Contradiction (LNC) says that no contradiction can be true. But what is a contradiction? And what would it take for a contradiction to be true? As Patrick Grim [5] has pointed out, a quick look at the literature will reveal a large menagerie of different interpretations of the basic terms and, consequently, of LNC. Grim actually identifies as many as 240 different options (on a conservative count), and I don't think there is any need to dwell further on the conceptual combinatorics that hides behind this familiar piece of logical nomenclature. I do, however, want to focus on one of the main ambiguities enumerated by Grim, one that seems to me to lie at the heart of the matter. And I want to offer an argument to the effect that on one way of resolving the ambiguity LNC is non-negotiable, but on another way it is perfectly plausible to suppose that LNC may, in some rather special and perhaps undesirable circumstances, fail to hold.

1. Two Notions of Contradiction

The ambiguity I have in mind is that which stems from the opposition between contradictions understood as individual statements (or propositions, or sentences), as for instance Ruth Marcus has it [14]:

(1) "A contradiction is the conjunction of a proposition and its denial,"

and contradictions understood as pairs of statements (propositions, sentences),[1] as in the definition given by Kalish, Montague, and Mar [8]:

(2) "A contradiction consists of a pair of sentences, one of which is the negation of the other."

[1] From now on I shall settle on 'statement', but this decision will be of no consequence.

Intuitively, the first type of contradiction arises if we assert and deny the same thing "in the same breath", whereas the second type of contradiction arises if we end up denying (perhaps unwittingly) something we have already asserted. We could plausibly generalize these formulations by construing in each case one of the two conjuncts, or statements, not as the negation of the other but as a conjunct or statement that is *equivalent* to the negation of the other. However, the notion of equivalence calls for a logic, and since I'm going to be concerned with the logical status of LNC it will be safer to stick to narrow formulations such as (1) and (2). Somewhat more formally, these can also be put thus:

(1') A contradiction is a statement of the form ϕ *and not-*ϕ.
(2') A contradiction is a pair of statements of the form ϕ and *not-*ϕ.

(where the italics serves the purpose of Quinean quotation). So the ambiguity arises from the fact that these two readings of 'contradiction' yield two corresponding readings of LNC:

(3) There is no circumstance in which a statement of the form ϕ *and not-*ϕ is true.
(4) There is no circumstance in which statements of the form ϕ and *not-*ϕ are (both) true.

Or, somewhat more formally:

(3') There is no circumstance X and no statement ϕ such that $X \Vdash \phi$ *and not-*ϕ.
(4') There is no circumstance X and no statement ϕ such that $X \Vdash \phi$ and $X \Vdash$ *not-*ϕ.

Let us call these the *collective* and the *distributive* formulations of LNC, respectively. Are these expressions of the same logical principle, or are they distinct?

In classical logic they are obviously equivalent. We are looking at two distinct formulations of what classically boils down to the same principle because classically truth and satisfaction commute with the truth-functional connectives, which is to say that a circumstance X verifies or satisfies a conjunction just in case it verifies or satisfies both conjuncts.[2] In other words, classically

[2] From now on, I shall confine myself to speaking of truth (broadly understood) rather than satisfaction. This is only to simplify things and will be of no consequence.

(3') and (4') are equivalent because of the following general equivalence (sometimes called the principle or rule of adjunction) governing the semantics of the connective ‘and’:

(5) $X \Vdash \phi$ *and* ψ if and only if $X \Vdash \phi$ and $X \Vdash \psi$.

If X is construed as a classical possible world, this equivalence is indisputable. And it is hardly a disputed equivalence even if the range of X includes worlds that are non-classical in some way or other, including impossible worlds that would violate LNC (for instance, a world inhabited by impossible objects such as Sylvan’s box [17], if such there be). A world in which a conjunction is true —it is often argued—*just is* a world in which the conjuncts are true, whether or not such truths comply with the laws of classical logic.

That LNC is to be taken as a principle about possible (or impossible) worlds is, however, another story. The intuition behind this and other logical principles is that they should provide some guidance as to what goes on in every conceivable circumstance, i.e., in every condition under which a statement might be said to be true or false; and this need not be cashed out in terms of worlds. Some prefer to speak of world *models* instead, or of *conceptions* of the world, and these in turn may be construed broadly enough so as to include, for example, fictional stories, pieces of discourse, belief sets, informational set-ups such as data-banks or knowledge-bases, and much more. In cases such as these the validity of (5) is no longer obvious, or so I shall argue; hence the distinction between collective and distributive understandings of LNC need not be empty. Indeed, in cases such as these the relevant notion of truth is itself liable to different characterizations. Some prefer to construe ‘$\Vdash$’ as expressing, not full-blooded truth, but rather some more metaphysically modest notion of *correctedness* or *acceptability* or *commitment* relative to X (and there is no obvious reason to suppose that logic should not be developed with such more modest notions in mind). Hence, not only can the collective and distributive understandings of LNC be distinguished; they can be distinguished even without rejecting classical semantics for truth *tout court*.

2. Worlds and Other Circumstances

To illustrate with an example, familiar from Belnap [2], suppose that a computer should be programmed so as to return ‘Yes’ to a query if and only if the relevant content has been explicitly entered in the computer’s data bank. If you enter ϕ the computer will say Yes to a query about ϕ, and if I enter ψ the computer will say Yes to a query about ψ, because each of us is independently trustworthy; yet the computer will reject the conjunction ϕ *and* ψ unless you are willing to agree with ψ or I am willing to agree with ϕ. In other words, the computer would only assent to a conjunction if both conjuncts come from the same source. If a state of the computer's data bank at any given time counts as a possible "circumstance", with '$\Vdash$' construed in the obvious way, then clearly this is a scenario in which (5) fails. In particular, the two versions of LNC will diverge, for my ψ could be the negation of your ϕ. Of course, we may want to supply our computer with a contradiction-checking device so as to prevent any circumstances of this sort from arising at all. But that is not to say that the computer could not work without the device. (Moreover, depending on the language used, there is no guarantee that the device could be effectively extended so as to detect *all* sorts of inconsistencies besides those involving pairs of explicitly contradictory statements.)

In a similar spirit, Jaskowski's discussive logic [12] is non-adjunctive, i.e., violates (5). If X is construed as reflecting the contents of a discussion involving two or more participants, so that the statements that hold in X are exactly those that are put forward by at least one of the participants, then there is no guarantee that the adjunction principle holds (from right to left). In particular, there is room for discordance. Two participants may contradict each other about the truth-value of a statement ϕ, but they need not contradict themselves. So again this could be a circumstance that complies with the collective form of LNC while violating the distributive form.

For one more example, familiar from the literature on the semantics of fiction, suppose we allow any sort of world-description to count as a circumstance. Such a description may involve discrepancies, as in *Don Quixote* (where there is discrepancy concerning the theft of Sancho Panza's ass[3]), or in the stories of *Sherlock Holmes* (where there is discrepancy concerning the position of Dr. Watson's war wound[4]), or even in the *Harry Potter* saga (where there is discrepancy concerning the order in which Harry's parents were murdered by

[3] In Don *Quixote*, I-xxiii, Gines steals Sancho's ass, but four pages later Sancho is riding it again. Cervantes comes back to this discrepancy and tries to fix it in the second part of the book (II-iv).

[4] We are told that Dr. Watson suffered a bullet wound during the Afghan campaign in which he participated. In *A Study in Scarlet*, this wound is said to be located in Watson's shoulder, but in *The Sign of Four* the wound is in his leg.

the evil Lord Voldemort[5]). But these discrepancies are best described as contradictions in the distributive sense rather than in the collective sense—as pairs of inadvertently contradictory statements rather than blatantly self-contradictory conjunctions. Hence the circumstances corresponding to these stories may be viewed as providing a violation to LNC in one sense but not in the other; they may be viewed as violating (4') (inadvertently) while complying with (3').

One might object that none of these cases should be given much credit. After all, one can easily insist that only worlds count as circumstances and treat all other cases as involving hidden propositional content of some sort. For example, it is true that a fictional story provides us with a context relative to which a statement might be said to be true or false; but this need not force us to construe the story as a genuine "circumstance". Rather—it could be argued—we could help ourselves with a suitable sentential operator that maps every statement ϕ to a corresponding statement of the form

(6) *According to S:* ϕ,

where S is the story in question (or the computer's data bank, or the record of a discussion, or what have you). Then the cases discussed above would allow us to question the following biconditional, where X is a genuine circumstance:

(7) $X \Vdash$ *According to S:* (ϕ *and* ψ) if and only if $X \Vdash$ *According to S:* ϕ *and according to S:* ψ.

And clearly this would not amount to questioning (5). 'According to S' is an operator that introduces an intensional context, on a par with 'Arthur said that' or 'Possibly', and one could insist that the question of whether such intensional contexts distribute over conjunction is to be settled case by case (depending on the sort of thing S is) and should be kept distinct from the question of whether (5) holds.[6]

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[5] The early books say that Harry's father died in an attempt to protect his child and wife, and that later Harry's mother was also killed. In *The Goblet of Fire*, when Harry forces the ghosts of all those killed by Voldemort and to eject themselves momentarily into the living world, the deaths are given in reversed order.

[6] Some such cases are particularly difficult to settle, of course, as illustrated by the debate concerning the deontic distinction between conflicts of obligation and logically incoherent obligations (i.e., between distributive and collective readings of the "ought implies can" principle). The relevance of these difficulties to the issue under discussion is one of the motivations for Schotch and Jennings's work on weakly aggregative modal logics [21, 23].

There is nothing wrong with this line of argument. None the less I don't think it can settle the matter, and for at least two reasons. First, I reckon that a systematic account of what should count as a genuine circumstance (in the relevant sense) is part of what it takes to define a logic, i.e., a theory of logical validity.[7] The pre-theoretic intuition is that an argument is logically valid if and only if its conclusion is true in every circumstance in which all its premisses are true,[8] and to make this precise one must come up with a precise characterization of the relevant notion of circumstance (along with an account of what it takes for a statement to be true in a given circumstance). But there is no a priori reason to assume that the range of options should be restricted to the realm of possible (and perhaps impossible) worlds. And it is hard to come up with a general assessment of LNC if we confine ourselves to a particular logic or family of logics. So it is certainly inappropriate to confine ourselves to a notion of circumstance that is restricted in the indicated way. Second, I reckon that the decision to treat a certain locution as belonging to the object language or to the metalanguage is itself part of what it takes to define a logic. We may decide to take the modal locution occurring in a statement such as

(8) *Possibly:* ϕ

as being part of the same (object) language to which the embedded statement ϕ belongs, as is customary in modal logic, or we may decide to push it up to the metalanguage and treat it as a semantical predicate of ϕ (a predicate to be cashed out in terms of quantification over worlds, for instance), as per Quine's "first grade of modal involvement" [19]. The choice is no philosophical routine and finds expression in a significantly different conception of modal logic. Likewise, we may decide to regard a locution of the form 'According to *S*' as being part of the object language, as per the line of argument under examination, or we may push it up to the metalanguage and treat it as a semantical predicate, as per the more liberal understanding of "circumstance" considered above. Again the choice is philosophically engaging and there is room for disagreement. But

[7] On this I align myself with Beall and Restall's "logical pluralism" [1]; see [30].

[8] Actually, it is unclear exactly how to phrase the pre-theoretic intuition. One could as well say that, intuitively, an argument is logically valid if and only if some of its premises is false in every circumstance in which its conclusion is false. If truth and falsehood are exhaustive and mutually exclusive, this coincides with the formulation given in the text; but if either truth-value gaps or truth-value gluts are admitted, then the two are distinct. Luckily, nothing here will depend on this ambiguity, so there is no need to bother.

the claim that *no* such locution should be treated as a semantical predicate *except when S is a world* is a strong claim that can hardly be taken for granted. Certainly an assessment of LNC—and of whether its collective and distributive readings are equivalent—should be viable independently of any such claim. If the claim is true, then (5) may well be true and so the two readings of LNC boil down to the same thing. But if the claim is false, then it would seem that (5) may fail and so the collective and distributive readings of LNC may be significantly distinct. .

3. Non-Contradiction and Excluded Middle

In fact, even if we stuck to the idea that the only admissible circumstances are worlds of some sort, the argument for (5) can hardly be that truth commutes with the truth-functional connectives. That is, that can hardly be the argument for (5) as soon as non-classical worlds come into the picture. If that were the case, then the rationale for (5) would also be a rationale for

(9) $X \Vdash$ *not*-ϕ if and only if not $X \Vdash \phi$.

Yet clearly (9) is controversial. Classically it holds. But as soon as X is allowed to range over circumstances in which a statement may fail to receive a definite truth-value, (9) seems to founder. For instance, if a statement ϕ suffers a truth-value gap then so does its negation (on most counts), hence the right-to-left direction of (9) may fail. Dually, if X is allowed to range over circumstances in which a statement may suffer a truth-value glut, then it is the left-to-right direction of (9) that is dubious: a circumstance X may verify the negation of a statement ϕ as well as ϕ itself. Neither of these possibilities depends on how exactly one construes the relevant circumstance X, e.g., on whether X is an incomplete or inconsistent story about the world or rather a world that is itself incomplete or inconsistent. Nevertheless, it is not at all unreasonable, or uncommon, to think that such circumstances violate the equivalence in (9) in one direction or the other. So why should (5) enjoy a different status in this regard? Why not consider the possibility that (5) (and consequently the equivalence between (3') and (4')) be rejected along similar lines as soon as we go beyond the scope of classical logic?

Consider also disjunction—a binary connective like conjunction. To say that truth commutes with this connective is to assert the following semantic equivalence:

(10) $X \Vdash \phi$ *or* ψ if and only if $X \Vdash \phi$ or $X \Vdash \psi$.

Again, this is classically valid and it is also valid in many non-classical logics. But there are also theories that reject (10). Supervaluationism, for instance, provides a semantics with truth-value gaps in which a disjunction can be true even if both disjuncts are indeterminate. As long as every admissible way of filling in the relevant gaps yields the same truth-value, a supervaluation assigns that value to the statement itself; so even if there is a circumstance X in which ϕ and ψ are both truth-valueless, the disjunction ϕ *or* ψ may still be true in X because it may be the case that every way of filling in the relevant gaps in X (every "completion" of X) verifies either ϕ or ψ. In particular, this is obviously the case if ψ is *not*-ϕ. Thus, supervaluationally it may be true that, say, a given color patch is either orange or red, even though the patch may be a borderline case of both orange and red; and it is true (in fact, logically true) that a given person is either tall or not tall, even though it may be indeterminate whether that person is tall (or not tall). This holds regardless of whether you take the relevant indeterminacy to be conceptual (e.g., a feature of our model of the world) or ontological (i.e., a feature of the world itself).[9] Regardless of how you construe X, if truth is supertruth then the equivalence in (10) may fail.

Indeed, as already Van Fraassen [26] pointed out, the failure of (10) shows that supervaluationism provides a means for distinguishing between the following two versions of the Law of Excluded Middle (LEM), which of course coincide in classical logic:

(11) For any circumstance X and any statement ϕ: $X \Vdash \phi$ *or not*-ϕ.
(12) For any circumstance X and any statement ϕ: $X \Vdash \phi$ or $X \Vdash$ *not*-ϕ.

Call these the *collective* and the *distributive* formulations of LEM, respectively. (The distributive form amounts to what is also known as the principle of Bivalence.) Then Van Fraassen's point was that supervaluationism validates only the collective formulation, not the distributive, as the case of the tallish person illustrates.[10]

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[9] Most supervaluationists would go with the former option (and I align myself with them [28]), but that is not a necessary feature of supervaluationism. See e.g. the (implicit) supervaluationism of Rescher and Brandon [20].

[10] Actually, the point can be traced back to Mehlberg [15], §29. Also, Van Fraassen's example involved non-denoting singular terms rather than vague predicates, but the same point applies. (See e.g. Fine [4].)

Some people find this distinction unintelligible. They would argue that the equivalence between the collective and the distributive reading follows directly from the so-called Equivalence Scheme for truth:

(13) It is true that ψ if and only if ψ.

For one can go from

(14) It is true that: ϕ or not-ϕ

to

(15) ϕ or not-ϕ

by applying the left-to-right direction of the Equivalence Scheme, hence to

(16) It is true that ϕ or it is true that not-ϕ

by applying (twice) the right-to-left direction.[11] However, it is clear that in the present context this argument would be question-begging. For, a more general rendering of (13) is

(13') For any circumstance X and any statement ϕ: $X \Vdash \phi$ if and only if ϕ_X

where 'ϕ_X' spells out the truth conditions for ϕ relative to X. Hence the general forms of (14)–(16) are:

(14') $X \Vdash \phi$ *or not*-ϕ
(15') $(\phi$ *or not*-$\phi)_X$
(16') $X \Vdash \phi$ or $X \Vdash$ *not*-ϕ.

And clearly the step from (15') to (16') is illegitimate on a supervaluational semantics. The correct step would be from (15') to

(17) For every admissible completion X' of X: $(\phi$ *or not*-$\phi)_{X'}$.

But this, as we have seen, does not imply

(18) Either for every admissible completion X' of X: $\phi_{X'}$, or for every admissible completion X' of X: *not*-$\phi_{X'}$.

[11] This line of argument may be found in Williamson [31] and Horwich [6], *inter alia*.

which is to say (by (13'))

> (19) Either for every admissible completion X' of X: $X' \Vdash \phi$, or for every admissible completion X' of X: $X' \Vdash$ *not*-ϕ,

which is the only legitimate reading of (16') afforded by supervaluationism. In other words, the objection to the distinction between collective and distributive readings of LEM is based on the assumption that truth commutes with the disjunction connective, which is precisely what is being denied by a supervaluational semantics.[12]

Now, LEM and LNC are often treated together, for they are dual. So a perfectly dual argument can be given in support of the distinction between the collective and distributive readings of LNC: if a semantics that violates the disjunction principle (10) allows one to distinguish between (11) and (12), it is plausible to suppose that a semantics violating the adjunction principle (5) should allow one to distinguish between (3') and (4'). In fact, such a semantics can naturally be constructed through a dualization of supervaluationism. Supervaluationism provides a way of dealing with *incomplete* circumstances by piggy-backing on their complete extensions. The intuition is that if the truth-value of a statement ϕ does not change as we consider different ways of disposing of the relevant gaps, then the gaps are not so relevant after all, at least as far as ϕ is concerned: if ϕ would be true no matter what, then let ϕ be true. (If, on the other hand, the truth-value of ϕ turns out to vary from extension to extension, then the gaps do appear to be relevant and ϕ cannot be assigned a definite truth-value.) In the case of inconsistent circumstances we could reason dually as follows. If a statement ϕ gets a certain truth-value on some admissible way of weeding out the relevant inconsistency, i.e., on some admissible consistent restriction of the given circumstance, then let ϕ have that value: after all, the circumstance is explicit about that. Otherwise don't give that value to ϕ. So if the value of ϕ changes as you go from one restriction to the next, then there is nothing we can do about it: the inconsistency appears to be irredeemable and ϕ will suffer a truth-value glut. But if the value of ϕ does not change as we consider different ways of disposing of the inconsistency, then the inconsistency turns out to be immaterial and ϕ may receive one and only one truth-value.

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[12] Nor is the distinction between the two readings a prerogative of supervaluationism. For a general discussion see DeVidi and Solomon [3].

Somewhat more formally, the idea can also be put thus.[13] The supervaluation registers the meet of all the admissible valuations, because an incomplete circumstance can itself be construed as the meet of its admissible complete extensions, or completions:

(20) $X \Vdash \phi$ if and only if $X' \Vdash \phi$ for every completion X' of X.

Dually, a "subvaluation" (as we may call it) will register the join of all the admissible valuations, because an inconsistent circumstance can itself be thought of as the join of its admissible consistent restrictions, or constrictions:

(21) $X \Vdash \phi$ if and only if $X' \Vdash \phi$ for some constriction X' of X.

If X is both incomplete and inconsistent, this pattern will have to be applied twice.[14] But if X is incomplete but not inconsistent, or vice versa, then the right-hand occurrence of '$\Vdash$' may well implement a perfectly classical set of truth conditions. In any event, it is clear that (21) provides a way of cashing out the intuition illustrated above with reference to such circumstances as fictional stories, data-bases, or discussive records. For if ϕ and ψ are overdeterminate (true and false), the conjunction ϕ *and* ψ may still be false (and only false) even if both ϕ and ψ are true (and also false), violating (5). In particular, if ψ is the negation of ϕ, then the conjunction comes out false (and only false) in every circumstance, i.e., logically false, even if both conjuncts are allowed to be true (and false).

To sum up, then, on this account the difference between the collective and the distributive forms of LNC turns out to be on a par with the difference between the collective and the distributive forms of LEM. It would not be an empty difference. And it would have repercussions on other logical principles as well, including principles governing the relation of logical consequence. For example, corresponding to the two readings of LNC and LEM one could also draw a difference between two readings of the principles known as *Ex falso quodlibet* (EFQ) and *Verum ex quodlibet* (VEQ), to the effect that contradictions logically imply everything and tautologies are implied by anything. On the collective reading these principles hold, just as in classical logic:

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[13] For details, complications, and generalizations I refer to Varzi [27, 28, 29]. See also Hyde [7] for an application to vagueness.

[14] In which order? As it turns out, the two options are not equivalent, but there is no need to worry about this here.

(22) ϕ *and not-*$\phi \models \Sigma$
(23) $\Sigma \models \phi$ *or not-*ϕ.

This follows directly from LNC and LEM. But the principles corresponding to the distributive reading,

(22') ϕ, *not-*$\phi \models \Sigma$
(23') $\Sigma \models \phi$, *not-*ϕ

may fail. (Here, 'Σ' ranges over arbitrary sets of statements, hence the implication relation '$\models$' is to be understood along the following lines:

(24) $\Sigma \models \Gamma$ if and only if, for every circumstance X, $X \Vdash \phi$ for all $\phi \in \Sigma$ only if $X \Vdash \phi$ for some $\phi \in \Gamma$.

I use this multiple-conclusion format to highlight the perfect duality between the two cases.[15])

4. The Truth-Functional Intuition

If this line of reasoning is taken seriously, then, the idea that LNC is ambiguous in an interesting way can be supported not only by considering different ways of specifying the basic notion of a circumstance, as in Section 2, but also by considering different ways of specifying the basic notion of truth. So, is this line of reasoning to be taken seriously?

I think there is still one objection against it that most people find decisive, and it has something to do with a certain intuition about the link between formal semantics and theory of meaning. Briefly, the objection is that any semantics that does not fully satisfy the equivalencies expressed by the adjunction principle (5), or by the corresponding disjunction principle (10), fails to do justice to the "meaning" of the conjunction and disjunction connectives as these are supposed to work in the English language. These equivalencies are non-

[15] Again, other notions of implication are possible, trading on the non-exhaustiveness and non-exclusiveness of truth and falsity (see note 8), but the main point holds regardless (Varzi [29]). It is also worth pointing out that if '$\models$' is cashed out in terms of other semantic features besides truth and falsity (e.g., in terms of Schotch and Jennings's levels of coherence [13]), then again the classical equivalence between the collective and distributive readings of EFQ and VEQ may be lost. In this sense, a broadly "preservationist" account of the consequence relation [22] would provide a short-cut to the conclusion of this section.

negotiable, it is argued, because they are meaning constitutive. Conjunction and disjunction are Boolean functions defined by certain truth tables; hence assignments of truth-values to a statement ϕ containing such connectives should be uniform functions of the truth-values of the sentential components of ϕ, which means that a truth-value should be assigned to ϕ if and only if the same value is assigned to every other statement of the same form whose components have the same values. If a semantics delivers a different account, so much the worse for the semantics. (Negation is a different story, it could be argued, because of its many meanings and uses, so don't worry about (9). Ditto for conditionals. But conjunction and disjunction are perfectly unambiguous in the relevant sense.)

This objection is particularly common in the case of disjunction and the supervaluational failure of (10). In his *Lectures on Truth* [9], for example, Saul Kripke has argued that supervaluationism sits very ill with the way we tend to respond to information conveyed by disjunctive statements. Someone says "ϕ or ψ" and we naturally ask: "Well—which one (if not both)?" Many people have echoed these misgivings. And similar objections have been raised against what I have called subvaluationism, too. For instance, Graham Priest and Richard Routley [18] regard the failure of adjunction as a sign that 'and' has departed from its normal interpretation: conjunction *just is* that connective whose truth conditions are fixed by (5).

In a way, this sort of objection can be dismissed on the grounds of its unfair appeal to intuition. Change of semantics, change of subject—says the objection. Fair enough. But who got the semantics right in the first place? Obviously the right semantics is the one that sits best with observable pragmatic phenomena: inclinations to assent or to dissent, and the like. Yet this is no easy game. If we are talking about a tallish person, we may feel uneasy in calling the statement 'This person is either tall or not tall' true because we wouldn't be able to say *which one*. When it comes to 'This person is both tall and not tall', however, our intuitions are much more unstable and range from mixed to strongly negative in spite of the underlying indeterminacy. Whence the difference? And how do phenomena of *this* sort sit with the truth-functional intuition?[16] Alternatively, the objection at issue is nothing more than an objection

[16] Nor is there any need to bring in truth-value gaps or gluts to make the point. As Kyburg [11] has emphasized, there is nothing incoherent in a circumstance in which we have many measurements, each of which is accurate enough to fall within the standard deviations of error, and yet we do not want to assent to their conjunction: "We can be certain that some of them is wrong!" A dual situation concerning disjunction is illustrated by the lottery paradox [10].

from the upper case letters, as Jamie Tappenden [25] calls it: "You say that *either* ϕ *or* ψ is true. So EITHER ϕ OR ψ [stamp the foot, bang the table] must be true!" ; "You say that ϕ AND ψ are true. So ϕ *and* ψ must be true!" Clearly this leaves us exactly where we were.

But never mind that. There is, I think, a deeper reply to this line of objection. For let us agree that conjunction and disjunction are indeed Boolean functions defined by the familiar truth tables. What follows from that? I think it follows that when we specify what counts as an admissible interpretation of the (object) language, we must rule out interpretations where 'and' and 'or' express something else than those Boolean functions. It does not, however, follow that (5) and (10) should hold, unless we make the extra assumption that there is a perfect homomorphism between a language and its interpretations. This point tends to be obfuscated by the fact that typically, as a matter of standard practice, the semantics of the logical operators is spelled out as being part and parcel of a recursive definition of truth: unlike the meaning of the other symbols (the "extra-logical" terms), the meaning of the logical operators is not specified by the structures used to interpret the language but rather fixed *indirectly* through a recursive definition of the truth-value of the statements in which they occur. It is imposed *ab initio* upon the entire semantic machinery. And such a recursive definition typically involves clauses that read exactly like (5) (the "truth conditions" for conjunction) or (10) (the "truth conditions" for disjunction). But this typical way of proceeding is misleading.

To appreciate this point, we have to bring in some background considerations concerning the status of logical terms in general. What is it that distinguishes the logical vocabulary from the extra-logical vocabulary? This is a difficult question, but this much is clear: the difference lies in the fact that the meaning of the logical terms is kept fixed, whereas the meaning of the extra-logical terms may vary. For example, an extra-logical term such as the predicate 'red' is characterized by a strong semantic variability: every interpretation that accords with its syntactic category is a legitimate interpretation for 'red' as far as logic goes; every such interpretation corresponds to some logically possible circumstance. By contrast, if we think that the equality predicate is a logical constant, then there is little room for semantic variability. Clearly we cannot just interpret it as the very same relation in all possible circumstances, for the relation designated by a binary predicate depends on the universe of discourse, and this can vary from circumstance to circumstance. We can, however, restrict the semantic variability of the equality predicate and "fix" its interpretation in the relevant sense by requiring that its extension be *always* the identity relation *restricted* to the relevant universe of discourse:

(25) In every circumstance X, the interpretation of the equality predicate '=' is the relation $\{\langle a, a\rangle : a \in \mathrm{U}_X\}$.

(where U_X is the universe of X). If "circumstance" is understood classically, of course, then certain plausible conditions on $\Vdash$ will ensure that (25) has the consequence:

(26) For every circumstance X and any pair of singular terms t_1 and t_2: $X \Vdash t_1 = t_2$ if and only if $I_X(t_1) = I_X(t_2)$

(where I_X is the interpretation function associated with X). For instance, this follows immediately if we adopt the familiar condition:

(27) If ϕ is an atomic statement of the form $Pt_1 \ldots t_n$, then $X \Vdash \phi$ if and only if $\langle I_X(t_1), \ldots, I_X(t_n)\rangle \in I_X(P)$.

Now, some people do exactly this when they spell out the semantics for a language with the equality predicate: they build (25) into the definition of an admissible circumstance (or "model") and they get (26) as a general corollary—whence the ordinary logical principles governing the equality predicate follow. Other people do it differently. They exploit the thought that if the meaning of equality is going to be "fixed" throughout, then there is no need to bring that explicitly into the interpretive machinery. So rather than using (25) as a constraint on what should count as an admissible circumstance, on this alternative account one uses (26) directly as a constraint on '$\Vdash$'. Both accounts are legitimate, because a logic is defined precisely by a specification of a certain set of constraints about these two notions: the notion of a (logically) admissible circumstance and the notion of truth in a circumstance. But there is a clear sense in which the second practice is conceptually contingent or dependent on the former: it is *because* we have (25) in the back of our mind that we can fix the meaning of equality indirectly via a clause such as (26). Being interpreted outside the interpretive machinery is not what distinguishes a logical term such as '=' from an extra-logical term such as 'red'.[17] That is something which is made

[17] This is the view commonly attributed to Tarski. (See e.g. Sher [24].) I try to articulate my disagreement with this view in my [30].

possible *by* the fact that '=' is selected as a logical term whereas 'red' is not, i.e., by the fact that the meaning of '=' is treated as constant (in the specified sense) whereas the meaning of 'red' is treated as variable.

And notice: if we did not agree on the relevant notion of a circumstance, or on the notion of truth, then the second option might not even be available. We might agree on (25) while disagreeing on (26). We might, for instance, agree that '=' stands for the identity relation and yet disagree on the truth-conditions of certain equality statements involving non-denoting terms, or vague terms, or ambiguous terms. Clearly that would not mean that one of us is attaching a non-standard meaning to the equality predicate. It would simply mean that we are drawing different consequences from the fact that we attach that meaning to that predicate. We would disagree on the logic of equality. (And it would be correct to say this precisely in so far as we agree on '=' being equality.)

Now, what about 'and' and 'or'? I think a perfectly similar story can (and ought to) be told. Of course, in this case we are talking about expressions that belong to a different syntactic category than equality. These expressions are connectives and so their semantic interpretation must be a function on the set of truth-values rather than a relation on the universe of discourse.[18] What are these truth-values? In principle they need not be fixed once and for all, but typically the set of truth-values is not allowed to vary from one circumstance to another. This amounts to a stipulation along the following lines:

(28) Every circumstance X has the same set of truth values T_X.

A classical logician would pick something like $\mathrm{T}_X = \{0,1\}$. A three-valued logician could go for $\mathrm{T}_X = \{0, .5, 1\}$; and a fuzzy logician might go for the continuum-valued set $\mathrm{T}_X = [0, 1]$. Once we have made up our minds, the set of truth-values is fixed and we can produce our definition of truth. For example, if X is a classical circumstance, we may agree to define truth thus:

(29) $X \Vdash \phi$ if and only if $\mathrm{V}_X(\phi) = 1$,

where V_X computes the truth table for ϕ.[19] So, now, when we say that 'and' and

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[18] One can also construe connectives as functions on sets of states of affairs, propositions, and much more. I will stick to Fregean truth-values for simplicity.

[19] The entries in the table would be partly guaranteed by (27) (which now implies that $\mathrm{V}_X(Pt_1 \ldots t_n)=1$ iff $\langle I_X(t_1), \ldots, I_X(t_n)\rangle \in I_X(P)$) and partly by whatever clauses fix the truth conditions for the non-truth-functional compounds in the language, e.g., quantified statements.

‘or’ are to be treated as truth-functions we mean to say that they have to be treated as functions on T_X. And to say that these are the usual truth-functions is to make a stipulation along the following lines:

(30) In every circumstance X, the extension of the connective ‘and’ is the function $\{\langle a, b, \min(a,b)\rangle: a, b \in T_X\}$.

(31) In every circumstance X, the extension of the connective ‘or’ is the function $\{\langle a, b, \max(a,b)\rangle: a, b \in T_X\}$.

Of course, these are not the only possibilities. For example, a fuzzy logician might prefer replacing (30) with

(30') In every circumstance X, the extension of the connective ‘and’ is the function $\{\langle a, b, a\cdot b\rangle: a, b \in T_X\}$.

Each stipulation corresponds to a certain way of fixing the meaning of the corresponding term, and each way of fixing the meaning will have certain consequences when it comes to matters of logical implication.

We thus come to the main point. Let us assume that we make the same stipulations concerning the meaning of ‘and’ and ‘or’, namely, that we agree on (30) and (31). And let us suppose that we also agree on the classical set of truth-values, taking $T_X = \{0,1\}$. *If* we also agree on the ordinary, classical way of understanding the notion of a circumstance and we agree on the ordinary, classical way of understanding ‘$\Vdash$’, then we certainly agree on (5) and (10) being a corollary of (30)–(31). That is, more precisely, we agree on the following being true:

(5') For any circumstance X and any pair of statements ϕ and ψ: $X \Vdash \phi$ *and* ψ if and only if $X \Vdash \phi$ and $X \Vdash \psi$.

(10') For any circumstance X and any pair of statements ϕ and ψ: $X \Vdash \phi$ *or* ψ if and only if $X \Vdash \phi$ or $X \Vdash \psi$.

We may agree so much on this that we may be inclined to think of (5') and (10') not as consequences of our background agreements but as a non-negotiable part of the machinery that we use to spell out our other agreements. Just as with equality, we may be inclined to specify our logical views not by using (30) and (31) as constraints on what should count as an admissible circumstance but rather by using (5') and (10') (hence (5) and (10) for short) as constraints that act directly on ‘$\Vdash$’. We might be inclined, that is, to pull our agreement out of the interpretive machinery and to build it into a recursive machinery that matches our choice of truth-functions. That is perfectly all right, precisely because we agree on both. But just as with equality, I submit that this alternative way of proceeding is conceptually contingent. It is because we have (30) and (31) in the back of our mind that *in practice* we can fix the meaning of ‘and’ and ‘or’ through such clauses as (5') and (10').[20]

Now suppose we *don’t* agree on what counts as an admissible circumstance. Or suppose we *don’t* agree on ‘$\Vdash$’, i.e., on what it takes for a statement to be true (or acceptable) under a given circumstance. Can we still agree that conjunction and disjunction have a certain meaning, namely, the meaning fixed by (30) and (31)? Of course we can. Does it follow that both of us will agree on (5') and (10')? Of course it doesn’t. If my notion of a circumstance is wider than yours, as in the examples of Section 2, or if my notion of truth is super- and subvaluational, as in the examples of Section 3, then (5') and (10') will not hold. Rather, in that case the logical properties of the conjunction and disjunction connectives would be captured by the following weaker facts (where ‘$\dashV$’ expresses falsehood):

(5'') For any circumstance X and any pair of statements ϕ and ψ:
- $X \Vdash \phi$ *and* ψ only if $X \Vdash \phi$ and $X \Vdash \psi$.
- $X \dashV \phi$ *and* ψ if $X \dashV \phi$ or $X \dashV \psi$.

(10'') For any circumstance X and any pair of statements ϕ and ψ:
- $X \Vdash \phi$ *or* ψ if $X \Vdash \phi$ or $X \Vdash \psi$.
- $X \dashV \phi$ *or* ψ only if $X \dashV \phi$ and $X \dashV \psi$.

And clearly these would not be good enough to allow me to cut a long story short and build the interpretation of the connectives into a recursive set of biconditionals. Does this mean that I would be attaching a different meaning to the connectives than you do? Again, the answer is—it doesn’t. It simply means that my other views (about the notion of circumstance and/or the notion of truth) would prevent me from drawing certain consequences from the fact that I attach that meaning to those connectives. It means that my logic of ‘and’ and ‘or’ would be different from yours, just as my logic of ‘=’ might be different from yours even if we agree on the meaning of ‘=’. It means, in particular, that while your theory implies both the collective and the distributive readings of LNC and LEM, or of EFQ and VEQ, my theory would only imply the collec-

[20] Admittedly, supporters of rule-following accounts of the meaning of logical constants will not like this. I’ll get back to this in the concluding section.

tive readings. But that is not to say I would have departed from the usual understanding of ‘and’ and ‘or’.

5. Conclusions

This line of argument is hardly going to convince anyone who has different views on the interplay between formal semantics and the theory of meaning—especially those who favor some sort of rule-following account of the meaning of logical constants. If the meaning of such expressions is fixed by their logical properties (by their “inferential role”, as some like to say[21]) then the primacy of explicit stipulations such as (30) and (31) dissolves. One would rather say that it is precisely principles such as (5') and (10'), or perhaps (5") and (10"), that take us close to the meaning of ‘and’ and ‘or’. To someone who holds this view I can only concede that any disagreement concerning such clauses is likely to entail a disagreement on the very meaning of those connectives. (Ditto for equality.) But then we are back to battles of intuitions and arguments from the upper-case letters. To the extent that the general picture outlined in the previous section is accepted, however, it seems to me that the truth-functional intuition about the meaning of the connectives is by no means incompatible with the rejection of the adjunction and disjunction principles (5) and (10). We can agree on the meaning of ‘and’ and ‘or’ while disagreeing on their logical properties. Hence, in particular, we may agree on the validity of LNC (or LEM) under the collective reading but not under the distributive reading. I therefore conclude that the difference between the two readings is not empty. It is, in fact, an important distinction that is likely to show up as soon as we get away from a certain standard, restricted way of understanding the notion of a possible circumstance and the corresponding notion of truth.[22]

References

[1] Beall, J. C., and Restall, G., ‘Logical Pluralism’, *Australasian Journal of Philosophy* 78 (2000): 475–493

[2] Belnap, N. D. Jr., ‘A Useful Four-Valued Logic’, in J. M. Dunn and G. Epstein (eds.), *Modern Uses of Multiple-Valued Logics*, Dordrecht: Reidel, 1977, pp. 8–37.

[21] I have in mind the sort of view inspired by the work of Prawitz [16], for instance.

[22] Thanks to two anonymous referees for helpful comments on an earlier draft.

[3] DeVidi, D., and Solomon, G., ‘On Confusions about Bivalence and Excluded Middle’, *Dialogue* 38 (1999): 785–799.

[4] Fine, K., ‘Vagueness, Truth, and Logic’, *Synthese* 30 (1975): 265–300.

[5] Grim, P., ‘What Is a Contradiction?’, this volume.

[6] Horwich, P., *Truth*, Oxford: Blackwell, 1990.

[7] Hyde, D., ‘From Heaps and Gaps to Heaps and Gluts’, *Mind* 106 (1997): 641–660.

[8] Jaśkowski, S., ‘Rachunek zdańdla systemów dedukcyjnych sprzecznych’, *Studia Societatis Scientiarum Torunensis, Sectio A* 1, 8 (1948): 55–77; English translation as ‘Propositional Calculus for Contradictory Deductive Systems’, *Studia Logica* 24 (1969): 143–157.

[9] Jennings, R. E., and Schotch, P. K., ‘The Preservation of Coherence’, *Studia Logica* 43 (1984): 89–106.

[10] Kalish, D., Montague, R., and Mar, G., *Logic: Techniques of Formal Reasoning*, New York: Harcourt Brace Jovanovich, 1980.

[11] Kripke, S., *Lectures on Truth*, unpublished ms., Princeton University, 1975.

[12] Kyburg, H. E., *Probability and the Logic of Rational Belief*, Middletown (CT): Wesleyan University Press, 1961.

[13] Kyburg, H. E., ‘The Rule of Adjunction and Reasonable Inference’, *Journal of Philosophy* 94 (1997): 109–125.

[14] Marcus, R., ‘Contradiction’, in T. Honderich (ed.), *The Oxford Companion to Philosophy*, Oxford: Oxford University Press, 1995.

[15] Mehlberg, H., *The Reach of Science*, Toronto: Toronto University Press, 1956.

[16] Prawitz, D., *Natural Deduction*, Stockholm: Almqvist and Wiksell, 1965.

[17] Priest, G., ‘Sylvan’s Box: A Short Story and Ten Morals’, *Notre Dame Journal of Formal Logic* 38 (1997): 573–582.

[18] Priest, G., and Routley, R., ‘Systems of Paraconsistent Logic’, in G. Priest, R. Routley, and J. Norman (eds.), *Paraconsistent Logic. Essays on the Inconsistent*, Munich: Philosophia-Verlag, 1989, pp. 151–186.

[19] Quine, W. V. O., ‘Three Grades of Modal Involvement,’ *Proceedings of the XIth International Congress of Philosophy*, Volume 14, Amsterdam: North-Holland, 1953, pp. 65–81.

[20] Rescher, N., and Brandom, R., *The Logic of Inconsistency. A Study in Non-Standard Possible-World Semantics and Ontology*, Oxford: Basil Blackwell, 1980.

[21] Schotch, P. K., and Jennings, R. E., ‘Modal Logic and the Theory of Modal Aggregation’, *Philosophia* 9 (1980): 265–278.

[22] Schotch, P. K., and Jennings, R. E., ‘Inference and Necessity’, *Journal of Philosophical Logic* 9 (1980): 327–340.

[23] Schotch, P. K., and Jennings, R. E., ‘Non-Kripkean Deontic Logic’, in R. Hilpinen (ed.), *New Studies in Deontic Logic*, Reidel: Dordrecht, 1981, pp. 149–162.

[24] Sher, G., *The Bounds of Logic. A Generalized Viewpoint*, Cambridge (MA): MIT Press, 1991.

[25] Tappenden, J., ‘The Liar and Sorites Paradoxes: Toward a Unified Treatment’, *Journal of Philosophy* 90 (1993): 551–577.
[26] Van Fraassen, B. C., ‘Singular Terms, Truth-Value Gaps, and Free Logic’, *Journal of Philosophy* 63 (1966): 481–495.
[27] Varzi, A. C., ‘Inconsistency Without Contradiction’, *Notre Dame Journal of Formal Logic* 38 (1997): 621–638.
[28] Varzi, A. C., *An Essay in Universal Semantics*, Dordrecht: Kluwer Academic Publishers, 1999.
[29] Varzi, A. C., ‘Supervaluationism and Paraconsistency’, in D. Batens et al. (eds.), *Frontiers in Paraconsistent Logic*, Baldock: Research Studies Press, 2000, pp. 279–297.
[30] Varzi, A. C., ‘On Logical Relativity’, *Philosophical Issues* 12 (2002): 197–219.
[31] Williamson, T., ‘Vagueness and Ignorance’, *Proceedings of the Aristotelian Society*, Suppl. 66 (1992): 145–162.