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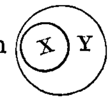
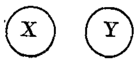
I. *On the Diagrammatic and Mechanical Representation of Propositions and Reasonings.* *By* J. VENN, *M.A.*, *Fellow and Lecturer in Moral Science, Caius College, Cambridge**.

SCHEMES of diagrammatic representation have been so familiarly introduced into logical treatises during the last century or so, that many readers, even of those who have made no professional study of logic, may be supposed to be acquainted with the general nature and object of such devices. Of these schemes one only, viz. that commonly called "Eulerian circles," has met with any general acceptance. A variety of others indeed have been proposed by ingenious and celebrated logicians, several of which would claim notice in a historical treatment of the subject; but they mostly do not seem to me to differ in any essential respect from that of Euler. They rest upon the same leading principle, and are subject all alike to the same restrictions and defects.

Euler's plan was first proposed by him† in his 'Letters to a German Princess,' in the part treating of logical principles and rules. What we here represent is, of course, the extent or scope of each term of the proposition. We draw two circles, and make them include or exclude or intersect one another, according as the classes denoted by the terms happen to stand in relation to one another in this respect. Thus "All

* Communicated by the Author.

† According to Drobisch and Ueberweg, this circular device had been already proposed by two previous writers, viz. C. Weise and J. C. Lange.

X is Y" is represented in the form  ; "No X is Y" is represented  . When two propositions are to be combined into a syllogism, *three* circles are of course thus introduced, the mutual relations of the first and third being determined by their separate relations to the second.

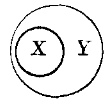
In spite of certain important and obvious recommendations about this plan, it seems to me to labour under two serious defects, which indeed prevent its effective employment except in certain special cases.

In the first place, then, it must be noticed that these diagrams do not naturally harmonize with the propositions of ordinary life or ordinary logic. To discuss this point fully would be somewhat out of place here; and as I have entered rather minutely into the question in a journal devoted to speculative inquiry*, I will confine myself to a very short statement. The point is this. The great bulk of the propositions which we commonly meet with are founded, and rightly founded, on an imperfect knowledge of the actual mutual relations of the implied classes to one another. When I say that all X is Y, I simply do not know, in many cases, whether the class X comprises the whole of Y or only a part of it. And even when I do know how the facts are, I may not intend to be explicit, but may purposely wish to use an expression which leaves this point uncertain. Now one very marked characteristic about these circular diagrams is that they forbid the natural expression of such uncertainty, and are therefore only directly applicable to a very small number of such propositions as we commonly meet with. Accordingly, if we resolve to make use of them, we must do one of three things. Either we must confine ourselves to propositions which are actually explicit in this respect, or in which the data are at hand to make them explicit—such as "X and Y are coextensive," "Some only of the X's are to be found amongst the Y's," and so forth; or we must feign such a knowledge where we have it not, which would of course be still more objectionable; or we must offer an *alternative choice* of diagrams, admitting frankly that, though one of these must be appropriate to the case in question, we cannot tell which it is. This third is the only legitimate course, and in the case of very simple propositions it does not lead to much intricacy; but when we have to combine groups of propositions, the

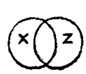
* 'Mind,' No. xix., July 1880.

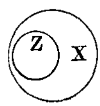
number of possible resultant alternatives would be very considerable.

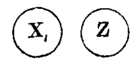
For instance, the proposition "All X is Y" needs *both* the diagrams, 



; for we cannot tell, from the mere verbal statement, whether there are any Y's which are not X. Similarly the proposition "Some X is not Z" needs *three* other diagrams,







(These five relations, it may be remarked, comprise all the possible ways in which two terms may stand to one another.) Hence the combination of the two given premises could not be adequately represented by less than six figures. If more premises, and more complicated ones (such as we shall presently proceed to illustrate), are introduced, the consequent confusion would be very serious. The fact is, as I have explained at length in the article above referred to, that the five distinct relations of classes to one another (viz. the inclusion of X in Y, their coextension, the inclusion of Y in X, their intersection, and their mutual exclusion), which are thus pictured by these circular diagrams, rest upon a totally distinct view as to the import of a proposition from that which underlies the statements of common life and common logic. The latter statements naturally fall into four forms—the universal and particular, affirmative and negative; and it is quite impossible to make the five divisions of the one scheme fit in harmoniously with the four of the other.

The second objection to which this scheme is obnoxious is of a more practical character; and viewed in that light it is, if any thing, of a still more serious character. It consists in the fact that we cannot readily break up a complicated problem into successive steps which can be taken independently. We have, in fact, to solve the problem first, by determining what are the actual mutual relations of the classes involved, and then to draw the circles to represent this final result; we cannot work step by step towards the conclusion by aid of our figures.

The extremely simple examples afforded by the syllogism do not bring out this difficulty; and it is consequently very apt to be overlooked. Take, for instance, the pair of propositions, "No Y is Z," "All X is Y." Here we have the relation of X to Y, and of Y to Z, given independently of one another; and this immensely simplifies the problem. We can think of each pair of circles without troubling ourselves about the other pair; we have nothing resembling *implicit* equations. But suppose that, on the other hand, we had a statement of the relation of X to Y and Z combined with others giving that of Y to Z and W, and, say, X to W, we should hardly know where to begin. Each statement being interlinked with the others, no one of them could be disentangled and represented separately. No doubt when the problem had been solved somehow, and a full determination secured of the mutual relations of the various classes, we could then set about undertaking to draw our circles. But this is a very different thing from working by help of the diagrams and employing them to aid our conceptions in the actual task of solution. The simple fact is that on this scheme, as already remarked, we have no means of exhibiting imperfect knowledge. What is exhibited is the *final outcome* of the relation, the actual exclusion or inclusion of the classes; and consequently we cannot represent our partial knowledge or the steps by which we attain to complete information. This defect comes out even in such a simple case as the ordinary disjunctive proposition "Every X is either Y or Z." Such a statement gives us no information as to the mutual relations of Y to Z; and therefore, since we have no means of marking by aid of our circles any thing but the actual relations of these classes, we should have to draw out a complete scheme of all the possibilities. This would demand, to begin with, five different figures displaying the five possible relations of Y to Z. We should then have to proceed to draw our X circle in each case, applying it as well as we could to each of these different figures. It will not need a moment's consideration to realize how tedious and complicated such a process would soon become when several class terms have thus to be combined.

We must therefore cast about for some new scheme of diagrammatic representation which shall be competent to indicate imperfect knowledge on our part; for this will at once enable us to appeal to it step by step in the process of working out our conclusions. I have never seen any hint at such a scheme, though the want seems so evident that one would suppose that something of the kind must have been proposed before. The one here offered may be said to underlie Boole's method *, and to be the appropriate diagrammatic

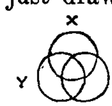
* I tried at first, as others have done, to represent the complicated propositions, there introduced, by the old plan; but the representations failed altogether to answer the desired purpose; and after some consideration I hit upon the plan here described.

representation for it. He makes no employment of diagrams himself, nor any suggestion for them.

One essential characteristic of Boole's method, as many readers of this article will probably know, is the complete subdivision of our field of inquiry into all the elementary classes which can possibly be yielded by combination of all the terms involved. Let there be two terms, X and Y; then we have to take account of the four subclasses, X that is Y, X that is not Y, Y that is not X, and what is neither X nor Y. Writing, for simplicity, $\bar{X}$ for not-X, the four classes are XY, $X\bar{Y}$, $\bar{X}Y$, $\bar{X}\bar{Y}$. Three class terms similarly yield eight subclasses, which admit of equally ready symbolic representation, and so on. Generally, if there be n classes involved in any given combination of logical premises there will be 2^n subclasses, every one of which must, somehow or other, be taken account of in any complete investigation of the problem.

This consideration seems to suggest a more hopeful scheme of diagrammatic representation. Whereas the Eulerian plan endeavoured at once and directly to represent *propositions*, or relations of class terms to one another, we shall find it best to begin by representing only *classes*, and then proceed to modify these in some way so as to make them indicate what our propositions have to say. How, then, shall we represent all the subclasses which two or more class terms can produce? Bear in mind that what we have to indicate is the successive duplication of the number of subdivisions produced by the introduction of every successive term, and we shall see our way to a very important departure from the Eulerian conception. All that we have to do is to draw our figures, say circles, so that each successive one which we introduce shall intersect once, and once only, all the subdivisions already existing, and we then have what may be called a general framework indicating every possible combination producible by the given class terms. This successive duplication of the number of subclasses was the essential characteristic when we were dealing with such symbols as X and Y. For suppose these two terms only involved, and there resulted the four minor classes indicated by XY, $X\bar{Y}$, $\bar{X}Y$, and $\bar{X}\bar{Y}$. Now suppose that a third term Z makes its appearance. This at once calls for a subdivision of each of these four into its Z and $\bar{Z}$ parts respectively. Thus XY is split up into XYZ and $XY\bar{Z}$, and so with the others, whence we get the eight subdivisions demanded. Provided our diagrams represent this characteristic clearly and unambiguously, they will do all that we can require of them.

The leading conception of this scheme is then simple enough; but it involves some consideration in order to decide upon the most effective and symmetrical plan of carrying it out. Up to three terms, indeed, there is but little opening for any difference; and as the departure from the familiar Eulerian plan has to be made from the very first, we will examine these simpler cases somewhat carefully. The diagram for two terms, then, is to be thus drawn :— 

On the common plan this would represent a *proposition*, and is, indeed, very commonly taken as illustrative of the proposition "Some X is Y."* With us it does not as yet represent a proposition at all, but only the framework into which propositions can be fitted; that is, it represents only the four combinations indicated by the letter-compounds XY, $X\bar{Y}$, $\bar{X}Y$, $\bar{X}\bar{Y}$. Now conceive that we have to reckon also with the presence, and consequently with the absence, of Z. We just draw a third circle intersecting the two above, thus, 

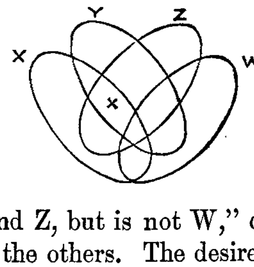
, and we have the eight compartments or classes which we need. The subdivisions thus produced correspond precisely with the letter-combinations. Quote one of these latter, and the appropriate class-division is ready to meet it; put a finger on any compartment, and the letter indication is unambiguous. Moreover both schemes, that of letters and that of spaces, agree in being mutually exclusive and collectively exhaustive in respect of all their elements. No one of the elements trespasses upon the ground of any other; and amongst them they account for all possibilities. Either scheme, therefore, may be taken as a fair representative of the other.

Beyond three terms circles fail us, since we cannot draw a fourth circle which shall intersect three others in the way required. But there is no theoretic difficulty in carrying out the scheme indefinitely. Of course any closed figure will do as well as a circle, since all that we demand of it, in order that it shall adequately represent the contents of a class, is that it shall have an inside and an outside, so as to indicate what does and what does not belong to the class. There is nothing to prevent us from going on for ever thus drawing successive figures, doubling the consequent number of subdivisions. The only objection is, that since diagrams are pri-

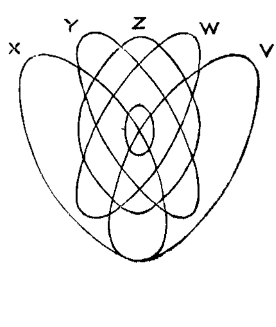
* It really takes, however, *three* common propositions to exhaust its significance; for the figure involves in addition the two statements "Some X is not Y," and "Some Y is not X."

marily meant to assist the eye and the mind by the intuitive nature of their evidence, any excessive complication entirely frustrates their main object.

For four terms the simplest and neatest figure seems to me to be one composed of four equal ellipses thus arranged:— It is obvious that we thus get the sixteen compartments that we want, counting, as usual, the outside of them all as one compartment. The eye can distinguish any one of them in a moment by following the outlines of the various component figures. Thus the one which is asterisked is instantly seen to be "X that is Y and Z, but is not W," or $XYZ\bar{W}$; and similarly with any of the others. The desired condition that these sixteen alternatives shall be mutually exclusive and collectively exhaustive, so as to represent all the component elements yielded by the four terms taken positively and negatively, is of course secured.



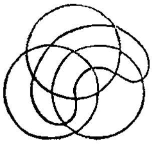
With five terms ellipses fail, at least in the above simple form. It would be quite possible to sketch out figures of a somewhat horse-shoe shape which should answer the purpose—that is, five of which should fulfil the condition of yielding the desired thirty-two distinctive and exhaustive compartments. For all practical purposes, however, any outline which is not very simple and easy to follow with the eye, fails entirely in its main purpose of affording intuitive and sensible illustration. What is wanted is that we should be able to distinguish and identify any assigned compartment in a moment, so as to see how it lies in respect of being inside and outside each of the principal component figures. For this purpose, when five class terms are introduced, I do not think that any arrangement will much surpass the following (the small ellipse in the centre is here to be reckoned as a piece of the *outside* of Z; *i. e.* its four component portions are inside of Y and W, but are no part of Z).

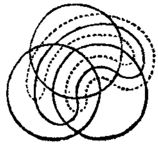


It must be admitted that such a diagram is not quite so simple to draw as one might wish it to be; but then we must remember what are the alternatives before any one who wishes to grapple effectively with five terms and all the thirty-two possibilities which they yield. He must either write down or in some way or other have set before him all those thirty-two compounds of which X Y Z W V is a sample; that is, he must contemplate the array produced by 160 letters. In comparison with most ways of doing *that*, the sketching out of such a figure is a pleasure, besides being far more expeditious; for, with a very little practice, any of the diagrams here offered might be drawn in but a minute fraction of the time requisite to write down all the letter-compounds. I can only say for myself that, after having for various purposes worked through hundreds of logical examples, I generally resort to diagrams of this description; it not only avoids a deal of unpleasant drudgery, but is also a valuable security against error and oversight. The way in which this last advantage is secured will be best seen presently, when we come to inquire how these diagrams are to be used to represent propositions as distinguished from mere terms or classes.

Beyond five terms it hardly seems as if diagrams offered much substantial help; but then we do not often have occasion to meddle with problems of a purely logical kind which involve such intricacies. If we did have such occasion, viz. to visualize the sixty-four compounds yielded by the six terms X, Y, Z, W, V, U, the best plan would probably be to take two of the above five-term figures—one for the U part and the other for the not-U part of all the other combinations. This would yield the desired distinctive sixty-four subdivisions, but, of course, it to some extent loses the advantage of the *coup d'œil* afforded by a single figure.

We have endeavoured above to employ only symmetrical figures, such as should not merely be an aid to the sense of sight, but should also be to some extent elegant in themselves. But for merely theoretical purposes the rule of formation would be very simple. It would merely be to begin by drawing any closed figure, and then proceed to draw others, subject to the one condition that each is to intersect once and once only all the existing subdivisions produced by those which had gone before. Proceeding thus we should naturally select circles as the simplest figures, so long as they would answer our purpose; that would be, up to three terms inclusive. The two successive modifications, aiming always at simplicity of figure, would then be naturally such as the following (the fifth figure is marked, for clearness, by a dotted line):—





A number of deductions will occur to the logical reader which it may be left to him to work out. Some of them may be just indicated. For instance, any two compartments between which we can communicate by crossing only one line, can differ by the affirmation and denial of one term only, *ex. gr.* $XYZW$ and $XY\bar{Z}W$. Accordingly, when two such are compounded, or, as we may say, "added" together, they may be simplified by the omission of such term; for the two together make up all XYW. Any compartments between which we can only communicate by crossing two boundaries, *ex. gr.* $XY\bar{Z}W$ and $X\bar{Y}ZW$, must differ in two respects; it would need *four* such compartments to admit of simplification, the simplification then resulting in the opportunity of dropping the reference to *two* terms; *ex. gr.* $XY\bar{Z}W$, $X\bar{Y}ZW$, $XYZW$, $X\bar{Y}\bar{Z}W$, taken together lead simply to XW. Many similar suggestions will present themselves.

So far, then, this diagrammatic scheme has only been described as representing terms or classes; we have now to see how it can be applied so as to represent propositions. Before doing this it will be necessary to indicate a certain view as to the Import of Propositions, because it is one which is not familiar or generally accepted, though it is very relevant and important for our present purpose. That view is briefly this —that every universal proposition, whether or not it be originally stated in a negative form, may be adequately represented by one or more *negations*. To give a complete justification of this view would involve a discussion which would be quite unsuitable to a general article like this; but a very few remarks will serve to explain, and to a considerable extent to justify it.

For instance, the common proposition "No X is Y," will be read as just denying the existence of the combination XY, and therefore needs but little alteration. The proposition "All X is Y" will be read as denying the combination "X that is not Y" or $X\bar{Y}$; and the destruction of that combination will here be regarded as its full import. "X is either Y or Z" will be considered fully accounted for when we have said that it denies "X that is neither Y nor Z" or $X\bar{Y}\bar{Z}$. "Every X that is not Y must be both Z and W" destroys the two combinations $X\bar{Y}\bar{Z}$ and $X\bar{Y}\bar{W}$, and so on. In a full exposition of the method here indicated, rules might conveniently be given for thus breaking up complex propositions into all the elementary denials which they implicitly contain; but the exercise of ordinary ingenuity will quite suffice thus to interpret any of the premises which we propose to take account of.

Another way of approaching the same question is by inquiring whether the various subdivisions in our diagram are to be considered as representing *classes*, or merely *compartments* into which classes may or may not have to be put. The latter view must be accepted as being the only one with which we can conveniently work. We may doubtless regard them as representing classes; but if we do so, we must keep in mind the proviso "if there be such a class of things in existence." And when this condition is insisted on, we appear to express our meaning best by saying that what our diagrammatic subdivisions (or, for that matter, the corresponding literal symbols) stand for are compartments which may or may not happen to be occupied.

One main reason for insisting upon this point is to be found in the impossibility of ascertaining, until we have fully analyzed our premises, whether or not any particular combination is possible. In the simple propositions of the common logic this difficulty hardly occurs; so that when we say "All X is Y," we take it for granted, or are apt to do so, that there must be both X's and Y's to be found. But if this proposition, or, still more, a complicated one of the same type, occurred as one of a group of premises, matters would be very different. We should then find that to *maintain* the existence of all the subjects and predicates, instead of merely *denying* the existence of the various combinations *destroyed* by them, would sadly hamper us in our interpretation of groups of premises*.

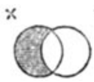
Take, for instance, the following group of premises, which are by no means of a very complicated nature:—

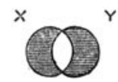
All X is either both Y and Z or not-Y,
All XY that is Z is also W,
No WX is YZ.

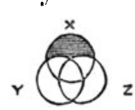
It would not be easy to detect, from mere contemplation of these data, that though they admit the possible existence of such classes as XZ and YZ, they deny that of the class XY. But since, as they stand, XY is the subject of one of them, we could not consistently admit such a conclusion unless we restricted the force of that second premise to what it *denies*, viz.

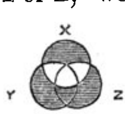
* I am not aware that it has ever been maintained that such a group of elementary denials is to be regarded as an *adequate* interpretation of these propositions. But it seems quite clear to me (on grounds too intricate to enter upon here) that this is the view which must be considered to underlie Boole's system, and, indeed, any general symbolic system of logic, if it is to be worked successfully.

by saying that it just destroys the class $XYZ\overline{W}$ or "X that is Y and Z but not W," and does nothing else*.

The method of employing the diagrams in order to express propositions will readily be understood. It is merely this:—Ascertain what each given proposition denies, and then put some kind of mark upon the corresponding partition in the figure. The most effective means of doing this is just to shade it out. For instance, the proposition "All X is Y" is interpreted to mean that there is no such class of things in existence as "X that is not-Y" or $X\overline{Y}$. All, then, that we have to do is to scratch out that subdivision in the two-circle figure, thus, 

. If we want to represent "All X is all Y," we take this as adding on another denial, viz. that of $\overline{X}Y$, and we proceed to scratch out that division also, thus, 

The main characteristic of this scheme, viz. the facility with which it enables us to express each separate accretion of knowledge, and so to break up any complicated group of data, and attack them in detail, will begin to show itself even in such a simple instance as this. On the common plan we should have to begin again with a new figure in each case respectively, viz. for "All X is Y," and "All X is all Y;" whereas here we use the same figure each time, merely modifying it in accordance with the new information. Or take the disjunctive "All X is either Y or Z." It is very seldom even attempted to represent this diagrammatically (and then, so far as I have seen, only if the alternatives are mutually exclusive); but it is readily enough exhibited when we regard it as merely extinguishing any X that is neither Y nor Z—thus, 

If to this were added the statement that "none but the X's are either Y or Z," we should then abolish the $\overline{X}Y$ and the $\overline{X}Z$, and have 

. Scratch out, again, the XYZ compart-

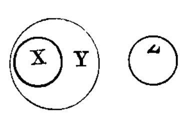
* Though this interpretation, however, of the import of propositions seems desirable for a really generalized system of logic, it is by no means necessary to adopt it in order to explain and justify the use of the diagrammatic method here proposed.

ment, and we have made our alternatives exclusive; *i. e.* the X is then Y or Z *only*.

Of course the same plan is easy to adopt with any number of premises. Our first data abolish, say, such and such classes. This is final; for, as already intimated, all the resultant elementary denials which our propositions yield must be regarded as absolute and unconditional. This first step then leaves the field open to any similar accession of knowledge from the next data; and so more classes are swept away. Thus we go on till all the data have had their fire; and the muster-roll at the end will show what classes may be taken as surviving. If, therefore, we simply shade out the compartments in our figure which have thus been successively proved to be empty, nothing is easier than to go on doing this till all the information yielded by the data is exhausted. In doing this it may, of course, often happen that some of the data wholly or partially go over the same ground as others. In that case, whichever of such data is considered after the other, finds its work already done for it entirely or in part; the class which we were going to mark for destruction is found to be already gone, and there is nothing to do so far as it is concerned.

As the syllogistic figures are the form of reasoning most familiar to ordinary readers, I will begin with one of them, though they are too simple to serve as effective examples. Take, for instance,

No Y is Z,
All X is Y,
∴ No X is Z.

This would commonly be exhibited thus, 

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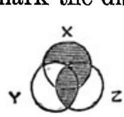
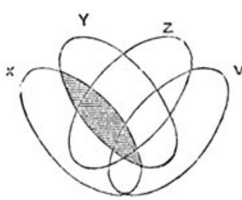
It is easy enough to do this; for in drawing our circles we have only to attend to two terms at a time, and consequently the relation of X to Z is readily detected; there is not any of that troublesome interconnexion of a number of terms simultaneously with one another which gives rise to the main perplexity in complicated problems. Accordingly such a simple example as this is not a very good one for illustrating the method now proposed; but, in order to mark the distinction, the figure to represent it is given, thus, 

In this case the one particular relation asked for, viz. that of X to Z, it must be admitted, is not made more obvious on our plan than on the old one. The superiority, if any, in such an example must rather be sought in the completeness of the pictorial information in other respects—as, for instance, that, of the four kinds of X which may have to be taken into consideration, one only, viz. the $XY\bar{Z}$, or the "X that is Y but is not Z," is left surviving. Similarly with the possibilities of Y and Z: the relative number of these, as compared with the actualities permitted by the data, are detected at a glance.

As a more suitable example consider the following—

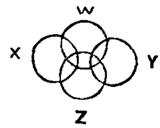
$$\begin{cases} \text{All X is either Y and Z, or not-Y,} \\ \text{If any XY is Z, then it is W,} \\ \text{No WX is YZ;} \end{cases}$$

and suppose we are asked to exhibit the relation of X and Y to one another as regards their inclusion and exclusion. The problem is essentially of the same kind as the syllogistic one; but we certainly could not draw the figures in the same offhand way we did there. Since there are four terms, we sketch the appropriate 4-ellipse figure, and then proceed to analyze the premises in order to see what classes are destroyed by them. The reader will readily see that the first premise annihilates all "XY which is not Z," or $XY\bar{Z}$; the second destroys "XYZ which is not W," or $XYZ\bar{W}$; and the third "WX which is YZ," or WXYZ. Shade out these three classes, and we see the resultant figure at once, viz.



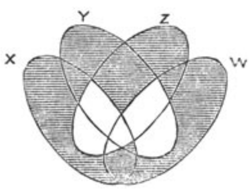
It is then evident that *all* XY has been thus made away with; that is, X and Y must be mutually exclusive, or, as it would commonly be thrown into propositional form, "No X is Y."

I will not say that it would be impossible to draw Eulerian circles to represent all this, just as we draw them to represent the various moods of the syllogism; but it would certainly be an extremely intricate and perplexing task to do so. This is mainly owing to the fact already alluded to, viz. that we cannot break the process up conveniently into a series of easy steps each of which shall be complete and accurate as far as it goes. But it should be understood that the failure of the older method is simply due to its attempted application to a somewhat more complicated set of data than those for which it was designed. But these data are really of the same kind as when we take the two propositions "All X is Y," "All Y is Z," and draw the customary figure. *When* the problem, however, *has been otherwise solved*, it is easy enough to draw a figure of the old-fashioned, or "inclusion-and-exclusion" kind, to represent the result, as follows,



; but one may safely assert that not many persons would have seen their way to drawing it at first hand for themselves*.

One main source of aid which diagrams can afford is worth noticing here. It is that sort of visual aid which is their especial function. Take the following problem:—"Every X is either Y or Z; every Y is either Z or W; every Z is either W or X; and every W is either X or Y: what further condition, if any, is needed in order to ensure that every XY shall be W?" It is readily seen that the first statement abolishes any X that is neither Y nor Z, and similarly with the others; so that the four abolished classes are $X\bar{Y}\bar{Z}$, $Y\bar{Z}\bar{W}$, $Z\bar{W}\bar{X}$, and $W\bar{X}\bar{Y}$. Shade them out in our diagram, and it stands thus:—



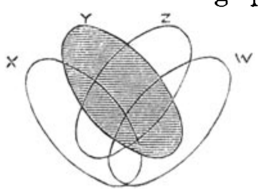
It is then obvious that, of the surviving component parts of XY, one only (viz. $XYZ\bar{W}$) is not W. If, then, this be destroyed, *all* XY will be W; that is, the necessary and sufficient condition is that "all XYZ is W."

* Even then we have said more in this figure than we are entitled to say. For instance, we have implied that there *is* some X which is W, and so forth. The other scheme does not thus commit us; for though the extinction of a class is final, its being let alone merely spares it conditionally. It holds its life subject to the sentence, it may be, of more premises to come. This must be noticed, as it is an important distinction between the customary plan and the one here proposed. The latter makes the distinction between rejection and non-rejection—such non-rejection being provisional, and not necessarily indicating ultimate acceptance. The former has to make the distinction between rejection and acceptance; for the circles must either intersect or not, and their non-intersection indicates the definite abandonment of the class common to both. Hence the practical impossibility of appealing to such diagrams for aid in representing complicated groups of propositions.

In the same way the implied total abolition of any one class is thus made extremely obvious. Take, for example, the following premises, and let us ask quite generally for any obvious conclusion which follows from them:—

Every Y is either X and not Z, or Z and not X;
Every WY is either both X and Z, or neither of the two;
All XY is either W or Z, and all YZ is either X or W.

It will be seen on reflection that these statements involve respectively the abolition of the following classes, viz.:—(1) of YXZ, $YX\bar{Z}$; (2) of $WYX\bar{Z}$ and $WY\bar{X}Z$; (3) of $XY\bar{W}\bar{Z}$, $YZ\bar{X}\bar{W}$. Shade out the corresponding compartments in the diagram, and it presents the following appearance—



It is then clear at a glance that the collective effect of the given premises is just to deny that there can be any such class of things as Y in existence, though they leave every one of the remaining eight combinations perfectly admissible. This, then, is the diagrammatic answer to the proposed question.

It will be easily seen that such methods as those here described readily lend themselves to mechanical performance. I have no high estimate myself of the interest or importance of what are sometimes called logical machines, and this on two grounds. In the first place, it is very seldom that intricate logical calculations are practically forced upon us; it is rather we who look about for complicated examples in order to illustrate our rules and methods. In this respect logical calculations stand in marked contrast with those of mathematics, where economical devices of any kind may subserve a really valuable purpose by enabling us to avoid otherwise inevitable labour. Moreover, in the second place, it does not seem to me that any contrivances at present known or likely to be discovered really deserve the name of logical machines. It is but a very small part of the entire process which goes to form a piece of reasoning which they are capable of performing. For, if we begin from the beginning, that process would involve four tolerably distinct steps. There is, first, the statement of our data in accurate logical language. This step deserves to be reckoned, since the variations of popular language are so multitudinous, and often so vague and ambiguous, that they may need careful consideration before they can be reduced to form. Then, secondly, we have to throw these statements into a form fit for the engine to work with—in this case the reduction of each proposition to its elementary denials. It would task the energies of a machine to deal at once, say, with all the premises employed even in the few examples here offered. Thirdly, there is the combination or further treatment of our premises after such reduction. Finally, the results have to be interpreted or read off. This last generally gives rise to much opening for skill and sagacity; for though in such examples as the last (in which one class, Y, was simply abolished) there is but one answer fairly before us, yet in most cases there are many ways of reading off the answer. It then becomes a question of judgment which of these is the simplest and best. For instance, in the last example but one, there are a quantity of alternative ways of reading off our conclusion; and until this is done the problem cannot be said to be solved. I cannot see that any machine can hope to help us except in the third of these steps; so that it seems very doubtful whether any thing of this sort really deserves the name of a logical engine.

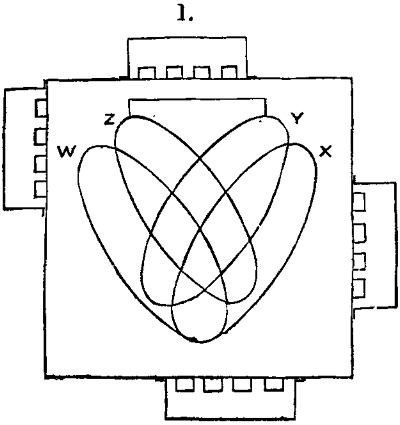
It may also be remarked that when we make appeal, as here, to the aid of diagrams, the additional help to be obtained by resort to any kind of mechanical contrivance is very slight indeed. So very little trouble is required to sketch out a fresh diagram for ourselves on each occasion, that it is really not worth while to get a machine to do any part of the work for us. Still as some persons have felt much interest in such attempts, it seemed worth while seeing how the thing could be effected here. There is the more reason for this, since the exact kind of aid afforded by mechanical appliances in reasoning, and the very limited range of such aid, do not seem to be generally appreciated.

For myself, if I wanted any help in constructing or employing a diagram, I should just have one of the three-, four-, or five-term figures made into a stamp; this would save a few minutes sometimes in drawing them; and we could then proceed to shade out or otherwise mark the requisite compartments. More help than this would be of very little avail. However, since this is not exactly what people understand by a logical machine, I have made two others, in order to give practical proof of feasibility.

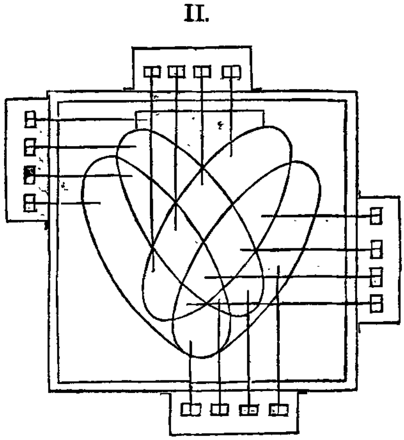
For instance, a plan somewhat analogous, I apprehend, to Prof. Jevons's *abacus* would be the following:—Have the desired diagram (say the five-term figure with its thirty-two compartments) drawn on paper and then pasted on to thin board. Cut out all the subdivisions by following the lines of the different figures, after the fashion of the children's maps which are put together in pieces. The corresponding step to shading out any compartment would then be the simple removal of the piece in question. We begin with all the pieces arranged together, and then pick out and remove those which represent the non-existent classes. When every one of the given premises has thus had its turn, the pieces left behind will indicate all the remaining combinations of terms which are consistent with the data. I have sometimes found it convenient, where the saving of a little time was an object, to use a contrivance of this kind. There is no reason to give a drawing of it, since any one of the figures we have hitherto employed may really be regarded as such a drawing.

Again, corresponding to Prof. Jevons's logical machine, the following contrivance may be described. I prefer to call it merely a logical-diagram machine, for the reasons already given; but I suppose that it would do very completely all that can be rationally expected of any logical machine. Certainly, as regards portability, nothing has been proposed to equal it, so far as I know; for though needlessly large as made by me, it is only between five and six inches square and three inches deep. It is intended to work for four terms; and the following figures will serve to show its construction:—

I.



II.



The first figure represents the upper surface of the instrument. It shows the diagram of four elipses, the small irregular compartment at the top of them being a representative part of the outside of all the four class-figures; that is, this compartment stands for what is neither X, Y, Z, nor W, or $\bar{X}\bar{Y}\bar{Z}\bar{W}$. The second figure represents a horizontal section through the middle of the instrument. Each of the ellipses here is, in fact, a section of an elliptical cylinder, these cylinders intersecting one another so as to yield sixteen compartments. Each compartment has a wooden plug half its height, which can move freely up and down in the compartment. When the machine is ready for use each plug stands flush with the surface, being retained there by a pin; we therefore have the appearance presented in fig. I. When we wish to represent the destruction of any class, all we have to do is slightly to draw out the appropriate pin (the pins of course are duly labelled, and will be found to be conveniently grouped), on which the plug in question drops to the bottom. This, of course, is equivalent to the shading of a subdivision in the plane diagram. As the plugs have to drop independently of one another, a certain number of them, it will be seen, have to have a slot cut in them, so as to play free from the pins belonging to other plugs. When the plugs have to be returned to their places at the top, all we have to do is to turn the instrument upside down, when they instantly fall back, and on pressing in the pins again they are retained in their place. The guards outside the pins are merely to prevent them from being drawn entirely out.

II. *On a simple Form of Saccharimeter. By* J. H. Poynting, *Fellow of Trinity College, Cambridge, Professor of Physics in Mason's College, Birmingham**.

THE general principle of the modification of the saccharimeter which I shall describe in this paper is well known, and has already been applied in the construction of several standard instruments, such as Jellett's and Laurent's. This principle consists in altering the pencil of rays proceeding from the polarizer in such a way that, instead of the whole pencil having the same plane of polarization, the planes of the two halves are slightly inclined to each other. The analyzer is therefore not able to darken the whole field of view at once. In one position of the analyzer the one half of the field is quite dark; in another position, slightly different, the other half is dark; while when the analyzer is halfway between these two positions, the two halves of the field are equally illuminated. This will be seen from the accompanying figure.

* Communicated by the Physical Society.