## Vassil VIDINSKY\*

## **Dynamical Interpretation of Leibniz's Continuum**

### Abstract

This dynamical interpretation of the continuum is based on a threefold perspective. First, detailed *differentiation* of all standard realms of Leibnizian Weltanschauung – (R real), (P phenomenal), (I ideal). Second, analysis of the *scope* of the Law of Continuity famously formulated by Leibniz and mapping it onto this (RPI) structure. Third, finding the *precise place* of dynamics and force in this (RPI) continuum.

These perspectives (taxonomical, legislative and junctional) if put together lead to a new understanding of monads' role; and they are not taken anymore as a discreet part of Leibnizian philosophy (as opposed to the ideal space and time), but as *dynamical continuum* incorporating in itself both contiguity and continuity. And in such a way they are both *neutralizing* and *preserving* the syncategorematic phenomenal infinity. The main point is that force can be applied both to perception and appetition of monads and by this we give the shortest Leibnizian answer to the Zeno's *Dichotomy* paradox – "force". But what is more important, such dynamical interpretation gives good schematic and systematic view of Leibnizian mature philosophy. And it appears (as expected) that the thread out of the Labyrinth of the Continuum is not only geometrical and physical, but metaphysical too.

#### Key Terms

Leibniz, Law of Continuity, Continuum, Force, Monad, Syncategorematic, Categorematic, Space, Time, Dynamics, Continuity, Contiguity, Zeno, Dichotomy, Metaphysics, Perception, Appetite, Infinity.

## Leibniz'deki Continuum'un Dinamik Bir Yorumu

## Özet

Continuum'un dinamik olan bu yorumu üçlü bir perspektife dayanmaktadır. İlkin Leibnizci Weltanschauung'un bütün standart gerçekliklerinin – (R gerçek), (P fenomenal), (I ideal)- detaylı olarak *ayrımlaştırılması*. İkinci olarak, Leibniz tarafından çok iyi bir biçimde formüle edilmiş olan süreklilik yasasının *kapsamının* bir analizi ve onun bu (RPI) yapısının üzerine yerleştirilmesi. Üçüncü

Asst. Prof. at the Faculty of Philosopy, University of Sofia, Bulgaria.

# Μω Καγθι 2008/10

olarak ise bu (RPI) continuum'unda dinamiğin ve gücün tam olarak yerinin belirlenmesi.

Eğer bu üç perspektif (sınıflayıcı, kural koyucu, ve birleştirici) bir araya getirilirse monadların rolünün yeni bir kavrayışına yol açar; ve monadlar artık sadece Leibnizci felsefenin sağduyulu parçaları olarak değil (ideal uzay ve zamana karşı olarak), fakat hem bitişikliği hem de sürekliliği kendi içine katan *dinamik bir continuum* olarak ele alınacaktır. Ve böylesi bir yolla onlar, syncategorematic fenomenal sonsuzluğu hem *etkisizleştirecek* hem de *muhafaza edecektir*. Temel husus, gücün, monadların algılarına ve iştihalarına uygulanabilir olmasıdır, ve bununla biz, Zenon'un *Dikotomi* paradoksuna Leibnizci cevabın en kısasını vermiş oluyoruz: "Güç". Fakat daha da önemli olan şey, böylesi dinamik bir yorumun, olgun Leibnizci felsefenin iyi şematik ve sistematik bir görünüşünü vermesidir. Ve görünüyor ki (beklenildiği gibi), Continuum Labirentinden çıkış yolu sadece geometrik ve fiziksel değil, fakat aynı zamanda metafizikseldir.

#### Anahtar Terimler

Leibniz, Süreklilik Yasası, Continuum, Güç, Monad, Syncategorematic, Categorematic, Uzay, Zaman, Dinamik, Süreklilik, Bitişiklik, Zenon, Dikotomi, Metafizik, Algı, İştiha, Sonsuzluk.

This paper is based on the recent scholarship and almost facet analysis by Richard Arthur, Glen Hartz, Jan Cover, Samuel Levey, Timothy Crockett, and François Duchesneau on Leibnizian continuum and philosophy. What I will try to do is a selective conceptual summary, also some corrections, and of course one further step – which I think changes the final perspective – a *dynamical interpretation* of the continuum. This whole analysis is actually based on a threefold perspective. Here are my main departing points:

First, we have to *differentiate* all the realms of Leibnizian Weltanschauung. Second we have to trace what exactly is the *scope* of the Law of Continuity famously formulated by Leibniz in 1704. And at the end we have to see what the *precise place* of dynamics in such continuum is.

But because all these perspectives (taxonomical, legislative and junctional) are interconnected within his philosophy (though changing until his mature thought), that's why my exposition will be rather systematical than chronological. On the other hand I will try to make my idea as clear as possible and I will artificially divide the paper in three sections; but keep in mind that systematicity of Leibniz's philosophy is the structural basis of my reading and the threefold paper division is propound only because it is more economical and neat.

## I. Weltanschauung – the three Leibnizian realms

As the old interpretation on Leibniz has put it "[he] splits the realm of the actual into two domains: the realm of *monads*, *the real world*, which forms *the object of study of metaphysics*; and the realm of the *things* of our everyday experience, *the phenomenal* 

## $m K\alpha\gamma \Omega$ 2008/10

*world*, which forms the object of study of the sciences in general, but pre-eminently of physics." [*italics added*]<sup>1</sup>. So we have the standard Rescher split: Monads [the real world presented by metaphysics] and Things [the phenomenal world presented by sciences]; which, I think, still governs modern interpretations but in a more subtle form.

But contemporary and recent scholarship is much more precise in its terminology and analysis of Leibniz's texts and thoughts. So we can advance one further division which separates Leibnizian Weltanschauung in three. In this paper I will try to show that the continuum problem is another crucial differentiation mark between these realms of the world: (1) monads; (2) things; (3) space and time. The full argument for separating them can be found in the detailed research by Hartz and  $\text{Cover}^2 - \text{I}$  will recall just 3 points from their conclusion:

(**R**) Monads have full and non-derivative *reality* (*R*).

(P) Only body – not every non-fundamental entity – is to be grounded on monads; and to be grounded means that you are not real as monads but *phenomenal* (P).

(I) Space and time are abstract entities (derived directly from phenomenal world and accessible by thought) and being abstract is a feature of *ideal (I)* things.

This (**R**)-(**P**)-(**I**) structure is hierarchical in such a way that every state is grounded on its left-standing realm. The analysis of Hartz and Cover covers mainly the connection between the phenomenal and ideal world, the monads are left out of the dynamical picture although they are important structural part in their paper too<sup>3</sup>.

So we have 3 different realms – (**R**) substantial, (**P**) quasi-substantial and (**I**) res mentalis<sup>4</sup>. Hartz and Cover interpretation is really strong and profound, but because the task they had defined was limited they didn't present a full analysis on the scope of the continuity, although it was central issue for the differentiation between (**P**) and (**I**). What I would like to do here is to map the Law of Continuity on this structure<sup>5</sup> – this will make the distinction clearer but will bring further questions.

<sup>&</sup>lt;sup>1</sup> Nicholas Rescher, *Leibniz: An Introduction to His Philosophy* (Totowa: Rowman and Littlefield, 1979) 65.

<sup>&</sup>lt;sup>2</sup> Glenn A. Hartz and J. A. Cover, "Space and Time in the Leibnizian Metaphysic," *Noûs* 22, no. 4 (1988).

<sup>&</sup>lt;sup>3</sup> Their concentration on P-I connection and differentiation is inevitable and logical – clarifying this relation is paper's main message and contribution. But Hartz and Cover wrote: "We believe it is possible to complete a 'reduction' of space and time to monads; doing so requires the use of purely Leibnizian materials to show how ideal space and time are related, *via* the intermediate level of phenomenal bodies, to features of monads. But that is a separate project." Well, partially I want to add several steps to this separate and more complete reading.

<sup>&</sup>lt;sup>4</sup> Hartz and Cover, "Space and Time in the Leibnizian Metaphysic," 503-04.

<sup>&</sup>lt;sup>5</sup> Timothy Crockett noticed that Leibniz is applying the Law of Continuity on these three levels, but still maintained that there are "two notions of continuity" following the twofold actual-ideal division – Timothy Crockett, "Continuity in Leibniz's Mature Metaphysics," *Philosophical Studies* 94, no. 1-2 (1999): 119-20. Compare his interpretation with Levey's historical analysis on the two types of continuity: "potentiality" and "connectedness" –

## π Καγοι 2008/10

## II. Law of Continuity in (P)-realm and (I)-realm

We have three different realms; but what is the implication of the legislative part of Leibnizian philosophy on them? I mean – what is the scope of the famous Law of Continuity on those realms and how does it work in ideal, phenomenal and real world? And if it is universal and covers everything is it one and the same law actually or it is limited and does have exceptions?

Let me start with the law itself, formulated in New Essays on the Human Understanding (1704); Leibniz states that there are nowhere leaps: "Nothing takes place suddenly, and it is one of my great and best confirmed axioms that *nature never makes leaps* [la nature ne fait jamais des sauts]. I call this the Law of Continuity"<sup>6</sup>.

So instead of stages we have continuous degrees; this is a straightforward and clear statement but the problem is that on the other hand we have exactly the opposite statements by the same Leibniz: "Matter is not continuous but discrete, and actually infinitely divided" (to De Volder, 11 Oct, 1705)<sup>7</sup>; "In actuals there is only discrete quantity" (to De Volder, 19 Jan, 1706)<sup>8</sup>. More than 10 years earlier Leibniz wrote to Foucher: "Thus I believe that there is no part of matter which is not, I do not say divisible, but actually divided; and that consequently the least particle ought to be considered as a world full of an infinity of different creatures"<sup>9</sup> and he said to Sophia that matter only appears to us to be continuum, just as does actual motion<sup>10</sup>; this is because matter is a discrete quantity and "the mass of bodies is actually divided in a determinate manner, and nothing in it is precisely continuous"<sup>11</sup> and things move from one state to the next closest state<sup>12</sup> and so on, and so on and even more...

Reading such contradictory passages Russell made his witty remark: "In spite of the law of continuity, Leibniz's philosophy may be described as a complete denial of the continuous"<sup>13</sup>. In a forthcoming paper Richard Arthur is showing that if you introduce the idea of syncategorematic infinity incompatibility between these various

Samuel Levey, "Matter and Two Concepts of Continuity in Leibniz," Philosophical Studies 94, no. 1-2 (1999).

Gottfried Leibniz, "Die Philosophischen Schriften Von Gottfried Wilhelm Leibniz," in OLMS Paperbacks (Hildesheim: Georg Olms Verlag, 1965), V, 49. Even earlier (April 3<sup>rd</sup>, 1699) he had already written to De Volder that no transition occurs by a leap - "nullam transitionem fieri per saltum" Leibniz, "Gp," II, 168.

<sup>&</sup>quot;Revera materia non continuum sed discretum est actu in infinitum divisum..." Leibniz, "Gp," II, 278.

<sup>&</sup>quot;In Actualibus non esse nissi dicretam Quantitatem..." Ibid., II, 282. "Ainsi je crois qu'il n'y a aucune partie de la nature qui ne soit, je ne dis pas divisible mais actuellement divisée, et par conséquent la moindre parcelle doit être considérée comme un monde plein d'une infinité de créatures différentes" Ibid., I, 416.

<sup>10</sup> "It is our imperfection and the shortcomings of our senses that make us conceive physical things as mathematical entities, in which there is indeterminacy" Ibid., VII, 563.

<sup>11</sup> Ibid., VII, 562.

<sup>12</sup> "d'un estat à l'autre prochain" Ibid., VII, 564.

<sup>13</sup> Bertrand Russell, A Critical Exposition of the Philosophy of Leibniz, with an Appendix of Leading Passages (Cambridge/New York: Cambridge University Press/Macmillan, 1900) 111.

## m Kαγ Ω1 2008/10

statements is only apparent. So Arthur is exonerating Leibniz from Russell's criticism and at the end of his analysis he says: "All naturally occurring transitions are *continuous* in that the difference between *neighboring* states is smaller than any assignable. This means not that there exists a least difference, but that for any assignable finite difference, there exists a smaller one. Thus there is a true *continuous* transition, even though the states themselves and all assignable differences between them are *actually discrete.*" [*italics added*]<sup>14</sup>.

But still it sounds like a puzzle, like a labyrinth – what kind of solution is Arthur's? Let me summarize everything in a few words to show more clearly the paradox: we have *different* statements in Leibniz which Russell claims that *contradict* each other and Arthur just puts them *together* – "continuous neighboring states" or "continuous transition of actually discrete states"<sup>15</sup>. Is this a real solution to Russell's attacks? And what is this 'syncategorematic' here for?

#### Categorematic vs. syncategorematic

The idea about syncategorematic interpretation of Leibniz is brought maybe for the first time by Ishiguro<sup>16</sup> in 70's and it is getting more and more powerful. But if we want to understand her or Arthur's defense and the validity of their interpretation we have to start from defining the notions of 'categorematic infinity' and 'syncategorematic infinity', i.e. Arthur's:

(C) *categorematic* – there exists some  $\langle y \rangle$  which is greater than any finite number  $\langle x \rangle$ ; or *there is a prime greater than every finite prime*.

(S) syncategorematic – for any finite number  $\langle x \rangle$  there is a number  $\langle y \rangle$  greater than it; or for every finite prime there is a greater prime<sup>17</sup>.

As Arthur points out there many cases where a mistake can be done by assuming that categorematic and syncategorematic infinity are one and the same; in other words

<sup>&</sup>lt;sup>14</sup> Richard T. W. Arthur, ""A Complete Denial of the Continuous"? Leibniz's Law of Continuity," in *Synthese* (forthcoming), 33.

<sup>&</sup>lt;sup>15</sup> We can trace the same issue in Crockett's proposal that there are things structurally continuous (S-continuous) and discrete (non M-continuous) but there's *nothing discontinuous* – Crockett, "Continuity in Leibniz's Mature Metaphysics," 132. Or even more literally paradoxical: discreteness of motion does not entail that motion is an aggregate of discrete states – Crockett, "Continuity in Leibniz's Mature Metaphysics," 133.

<sup>&</sup>lt;sup>16</sup> The base for Arthur's interpretation is the parallel which he makes between Leibniz's contradictory remarks about *continuity* and identical contradictory remarks on the *infinitesimals*. Arthur cites Ishiguro because she already had showed that the solution about the later contradictions lies in differentiation between syncategorematic and categorematic infinity, so Arthur tries to make a parallel solution: "Just as an actual infinity of terms can be understood syncategorematically as more terms than can be assigned a number, without there being any infinite numbers, so too the infinitely small can be given a syncategorematic interpretation by means of the Law of Continuity, without there existing any actual infinitesimals" – Arthur, ""A Complete Denial of the Continuous"? Leibniz's Law of Continuity," 33.

<sup>&</sup>lt;sup>17</sup> Ibid., 7-8.

it's a fallacy to say that (C) "there is a prime greater than every finite prime" = (S) "for every finite prime there is a greater prime". And the fallacy is quantifier shift fallacy<sup>18</sup>.

The point is almost clear, but I want to make a small correction and to offer a visual metaphor: because in (S) we have Archimedean axiom (as Leibniz knew), so we don't have biggest or smallest number and as a result the infinity is not number at all, that's why we can picture this by "ellipsis": [...]. And (C) is not that we have greater number, as Arthur formulated it, but we have greatest, so we can picture it as "full stop" [.] – and it is now getting clear why (S) can be treated as infinity and why (C) is not so suitable for Leibniz... Of course I have to be more precise in this case and write "not so suitable for Leibniz after 1676"; because before 1676 he made several attempts to grasp the continuum through categorematic infinity, using indivisible points, unassignable gaps, infinitesimal lines... The difference between (C) and (S) will be even clearer if we compare them in a simple scheme.

## Early categorematic solutions

In another very interesting forthcoming paper<sup>19</sup> Arthur traces the elusive development of Leibniz's early thought on the status of the actually infinitely small in relation to the continuum. He distinguishes three different categorematic stages prior to 1676<sup>20</sup>. From conceptual point of view they can be regarded not only as stages (which are abandoned one by one by Leibniz) but also as different perspectives towards the continuum problem. I will only summarize them in a scheme, because they do not have direct connection with my dynamical interpretation:

#### Leibniz's categorematic solutions. Mainly based on Arthur's paper

#### Table 01

Level	A. Metaphysical	B. Physical	C. Mathematical
Period	Pre-1670	1670-71	1672-75
Common ground	Francisco Arriaga, Kalam;	Hobbes	Sextus Empiricus, Hobbes
Main notions	Void (quietulas; esse nihil)	Parts (partes); Endeavor (conatus)	Lines (linea); Endeavor (conatus)
Arthur's definition	the continuum consists of assignable points separated by unassignable <b>gaps;</b>	the continuum is composed of an infinity of indivisible points, or <b>parts</b> smaller than any assignable, with no gaps between them;	a continuous line is composed of infinitely many infinitesimal <b>lines</b> , each of which is divisible and proportional to an element (conatus) of a generating motion at an instant;

<sup>&</sup>lt;sup>18</sup> It is a very original and clear difference, Ibid., 8.

<sup>&</sup>lt;sup>19</sup> "From Actuals To Fictions: Four Phases in Leibniz's Early Thought On Infinitesimals" will appear in *Studia Leibnitiana*.

Another interesting analysis on Leibniz's changing thought during these years is Levey, "Matter and Two Concepts of Continuity in Leibniz".

# Μ Καγθι 2008/10

Explanation	(1) Continuity of motion is interrupted by unassignable intervals of rest – times smaller than any given.	(1) Continuum is composed of infinitely small indivisibles, defined as parts which have magnitude, but no extension;	<ol> <li>Continuum is composed of infinitely small extended indivisibles, defined as parts which have magnitude proportional to the conatus of the generating motion;</li> </ol>
	(2) Bodies are continually (re)created.	(2) Continuity of motion is established through its composition out of conatus or endeavours	
		(3) Leibniz attempts to	
		distinguish minima (having zero magnitude) from indivisibles (having zero extension);	
Leibniz's argument		Inverted version of Zeno's <i>Dichotomy</i> argument	Diagonal paradox by Sextus Empiricus
Sources	De rationibus motus (1669-70) Letter to Thomasius (April 30 <sup>th</sup> , 1669)	Theoria Motus Abstracti (1671)	De minimo et maximo (1672-73)

As we can see Leibniz made several attempts to solve the continuum puzzle within itself, looking for different types of *compositional indivisibles*. And finally he couldn't find any categoremacity in this phenomenal world, which immediately made it "dependent on". This is my first step towards the dynamical interpretation of the continuum.

## Final syncategorematic solution

After 1676 Leibniz realized that the mathematics cannot provide categoremacity too (for any phenomenal realm), because there is no such thing as infinite number; and by number he meant something that can unite multiplicity. So here is the analogous scheme about the syncategorematic solution proposed by Leibniz.

# Μω Καγθι 2008/10

#### *Leibniz's syncategorematic solution. Mainly based on Arthur's paper*

Table 02

Level	A. Mathematical	
Period	After 1676	
Common ground	Archimedes	
Main notions	Infinitesimals are fictions (fictionem)	
Arthur's definition	"infinitesimals are <b>fictitious entities</b> , which may be used as compendia loquendi to abbreviate mathematical reasoning; they serve as a shorthand for the fact that finite variable quantities may be taken as small as desired, and so small that the resulting error falls within any preset margin of error"	
Explanation	(1) If infinitesimals are fictitious entities that mean that you cannot use the ideal mathematical realm as continuum <i>unity</i> . Infinitesimals are just abstractions which cannot unite physical aggregates.	
	(2) Our phenomenal realm cannot be dependent on our ideal realm.	
Leibniz's argument	No infinite number or infinite whole	
Sources	Pacidius Philalethi (1676)	
	Numeri infiniti (1676)	

This was Leibniz's final solution about geometrization of the continuum. That means – mathematics cannot provide the needed independence (categorematicity) for our phenomenal realm. This will lead us closer to his final systematical (neither purely idealist, nor materialist) account. But let me go ahead and deep in this syncategorematic solution.

## Syncategorematic continuum vs. Ideal continuum

Quite often syncategorematic infinity is defined as *potential* and categorematic infinity as *actual*. Exactly here lies the important change and paradox which we are revealing in Leibniz, because he is talking about *actual syncategorematic infinity*. What does this concept mean?

On one hand it is actual and for Leibniz that meant finite; because there is no such thing as infinite number (numbers are either even or odd -1, 2, 3, 4...)<sup>21</sup>; but on the other hand it is syncategorematical, which means infinite (there is always more to count and there are everywhere middle terms). So what we have here is exactly what both Russell and Arthur noticed from different perspectives – we have strange mixture

<sup>&</sup>lt;sup>21</sup> "... infinity, that is to say the accumulation of an infinite number of substances, is, properly speaking, not a whole any more than infinite number itself, whereof one cannot say whether it is even or uneven." *Theodicy* §195, Leibniz, "Gp," VI, 232.

between actual division and infinity of the division. But do we have *real continuum* than? Russell will say no, Arthur, I think, will say yes. But I claim that Russell would be right though he didn't understood Leibniz's point and Arthur would be partially wrong though he got the path out of the labyrinth. Here is my explanation of this obscure answer:

The phenomenal world is discrete and Leibniz said it many-many times - every interpretation starts and should start with this<sup>22</sup>. But on the other hand between every two things and states there are always more and more. Than he added that the margin of error between "continuous" circle and infinitely-sided "discrete" body is almost null. But this is not ontology anymore; it is part of his epistemology and of course the error is null in exactly the same way as matter *appears* to be continuous. The discreetness is so dense that the error between continuous (circle) and non-continuous (body) is unassignable. We can check another interesting example: "we know that a given ellipse approaches a parabola as closely as desired, so that when the second focus of the ellipse is removed far enough away from the first focus, the difference between the ellipse and the parabola becomes less than any given difference, since then the radii from that distant focus differ from parallel lines by an amount as small as desired"<sup>23</sup>. So when Arthur and Leibniz talk about assignability they are only within the discourse of epistemology. But from ontological perspective there is discreetness to infinity in the matter - the more it is actually divided the less it is really continuous. The phenomenal world is dense contiguum and not real continuum – "Contiguous things are those between which there is no distance"<sup>24</sup>. We have only syncategorematicity (infinite actual density) which is different from the continuum of space and time where we don't have any discreetness only the whole itself and the division in this ideal realm would be derivative and possible: "But space and time taken together constitute the order of possibilities of the one entire universe, so that these orders - space and time - relate not only to what actually is but also to anything that could be put in its place, just as numbers are indifferent to the *things which can be enumerated* 

So, I think we have to keep the difference between  $(\mathbf{P})$ -realm continuum and  $(\mathbf{I})$ -realm continuum in order to understand further distinctions between the phenomenal

<sup>&</sup>lt;sup>22</sup> I am not convinced that we have to make such artificial differentiation as Crockett did: discreetness (different things) is not discontinuity (having gaps) – Crockett, "Continuity in Leibniz's Mature Metaphysics.". Especially if having such remarks by Leibniz: "in order to have a variety of boundaries arising in matter a *discontinuity of the parts is necessary*" [*italics added*] – Gottfried Leibniz, "Gottfried Wilhelm Leibniz: Sämtliche Schriften Und Briefe," (Darmstadt/Leipzig: Otto Reichl Verlag, 1923-...), VI-2, 435. And one comment from Levey: "[matter] is not continuous but discrete; its parts are strictly discontinuous" – Levey, "Matter and Two Concepts of Continuity in Leibniz," 83-84.

<sup>&</sup>lt;sup>23</sup> Leibniz, "Ag," VI-4, 2032.

<sup>&</sup>lt;sup>24</sup> Ibid., VI-3, 94; translation in – Gottfried Leibniz, *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-1686*, ed. Daniel Garber and Robert C. Sleigh, trans. Richard T. W. Arthur, *The Yale Leibniz* (Yale: Yale University Press, 2001) 19.

<sup>&</sup>lt;sup>25</sup> Leibniz, "Gp," IV, 568.

## $m K\alpha\gamma \Omega$ 2008/10

and ideal, between matter and space, between motion and time, between parts and the whole  $^{26}$ ...

Let's make a quick summary: the Law of Continuity works within phenomenal world in such a way that it *manifests itself* as syncategorematical where contiguous approaches continuum (and the differentiating error between them is null) but still there is a difference between this phenomenal world and the realm of abstract space and time. The difference is smaller than any assignable, says Leibniz, but from fundamental level we can state the difference with his own words: "In actual, single terms are prior to aggregates, in ideals the whole is prior to the part" (to Des Bosses, 31 July, 1709)<sup>27</sup>. Of course you cannot assign such a qualitative difference, but you can state it. So what we see are syncategorematically infinite aggregates, not wholes, and aggregates are fundamentally discrete (though actually infinitely divided and approaching continuity). Such an unassignable error is quite close to Leibniz's identity of indiscernibles. We can use now this famous principle as a test-question: is it one and the same case to have a syncategorematically-sided body and continuous circle? If we talk about quantity – yes. But if we talk about quality – no. One thing can *approach* the other only if we keep this difference; and only than abstract mathematics can *measure* our phenomenal world. And when we think about fundamentals is not about usability and errors, but it is about priority: "Matter is not continuous but discrete, and actually infinitely divided, though no assignable part of space is without matter. But space, like time, is something not substantial, but ideal, and consists in possibilities, or in an order of coexistents that is in some way possible. And thus there are no divisions in it but such as are made by the mind, and the part is posterior to the whole. In real things, on the contrary, units are prior to the multitude and multitudes exist only through units" [*italics added*]<sup>28</sup>.

Than why dealing so much with phenomenal if we have an ideal realm? Because *extension is prior to space, as duration is prior to time.* Space and time derive from phenomenal world and the (**P**)-realm is the foundation of the (**I**)-realm. It seems that syncategorematic continuum (parts prior the whole) is the foundation of abstract continuum (whole prior the parts)<sup>29</sup>. How is it possible?

## *Syncategorematic infinity – a well-founded phenomenon*

Before going to junctional part of this paper let me be clear on one more point – the division (or cut) *does not produce* this phenomenal world; the division is only a

<sup>&</sup>lt;sup>26</sup> Jus remember that space and time are "are *perfectly* uniform and *arbitrarily* divisible" [*italics added*] – Hartz and Cover, "Space and Time in the Leibnizian Metaphysic," 499.

 <sup>&</sup>quot;In actualibus simplicia sunt anteriora aggregatis, in idealibus totum est prius parte" – Leibniz, "Gp," II, 379.
 28

<sup>&</sup>lt;sup>28</sup> "Revera materia non continuum sed discretum est actu in infinitum divisum, etsi nulla pars spatii assignabilis materia vacet. At spatium, ut tempus, non substantiale est quiddam, sed ideale, et in possibilitatibus seu ordine coexistentium utcunque possibili consistit. Itaque nullae ibi divisiones nisi quas mens facit, et pars toto posterior est. Contra in realibus unitates multitudine sunt priores, nec existunt multitudines nisi per unitates" – Ibid., II, 278-79.

<sup>&</sup>lt;sup>29</sup> If we put it this way it is clear that something is missing – both in epistemological and in ontological domain. And what is missing is the real world, the (**R**)-realm.

manifestation of its infinity and a differentiation between the first and second matter. But what makes or produces the phenomenal realm is *the connection* of its infinite parts, which put together make aggregates. So if left alone the syncategorematic infinity would lead to dissociation. And here we can see once more the difference between the (I)-realm and (P)-realm. In the abstract realm there is no dissociation, because we have the whole as its basis and the division (or the cut) just discloses its a priori continuity. Leibniz even wrote that in ideal realm "the notion of the whole is simpler than that of fractions, and precedes it" [*italics added*]<sup>30</sup>. But for phenomenal realm we need something else which will neutralize and preserve its syncategorematic infinite division - it cannot be (an abstract) space, because Leibniz is relationist and do not use space and time as substratum. Concerning this frenetic division (that I claim would lead to dissociation) Leibniz wrote to Foucher (1693): "Thus I believe that there is no part of matter which is not, I do not say divisible, but actually divided; and that consequently the least particle ought to be considered as a world full of infinity of different creatures"<sup>31</sup>. This thought is already conceptualized in Theory of concrete motion (1670-1671) where he famously stated that every atom will be of infinite species [quaelibet atomus erit infinitarum specierum] and there are worlds within worlds to infinity [mundi in mundis in infinitum]<sup>32</sup> and within every fold in the fold there is another endlessly folded world. What makes this split (approaching continuum) tenable? How it doesn't fall apart - I mean both the phenomenal world and Leibniz's conception?

Or we can state it as Stuart Brown: "[the problem is] how anything that is extended in space or time can be real if each of its parts is further divisible *ad infinitum*"<sup>33</sup>. If we are looking for reality we have to switch the realm. So its time to analyze the real world, because if we want to make consistent continuum theory we have to apply the Law of Continuity to all the realms of the Leibnizian Weltanschauung.

#### **III.** Dynamics – the (R)-realm

Up to now we left (**R**)-world somehow out of the picture. So what are the characteristics of this realm? Immediate, short answer: these are the characteristics of the monads themselves<sup>34</sup>.

<sup>&</sup>lt;sup>30</sup> Letter to Electress Sophie (31 Oct, 1705) – Leibniz, "Gp," VII, 562.

<sup>&</sup>lt;sup>31</sup> Ainsi je crois qu'il n'y a aucune partie de la matiere qui ne soit, je ne dis pas divisible, mais actuellement divisée, et par consequent, la moindre particelle doit estre considerée comme un monde plein d'une infinité de creatures differentes – Ibid., I, 416. Or: "There is an infinity of creatures in the smallest particle of matter, because of the actual division of the continuum to infinity." *Theodicy* §195, Leibniz, "Gp," VI, 232.

<sup>&</sup>lt;sup>32</sup> Leibniz, "Gp," IV, 210. The influence by Robert Hook's *Micrographia* is obvious.

<sup>&</sup>lt;sup>33</sup> Stuart Brown, "The Seventeenth-Century Intellectual Background," in *The Cambridge Companion to Leibniz*, ed. Nicholas Jolley (Cambridge: Cambridge University Press, 1995), 51.

<sup>&</sup>lt;sup>34</sup> As Garber showed there are at least two metaphysical strains in Leibniz ontology. I will stick to the more popular one up to now, the monadological – Daniel Garber, "Leibniz: Physics and

# m Kαγρι 2008/10

#### Monads - step out of the Labyrinth, step into the Continuum

As Arthur have stated: "In this wider sense, the continuum problem is: what (if any) are the first elements of things and their motions?"<sup>35</sup> So let me go through these first elements and Leibniz's *Monadology*. Because it is a well-known territory I will make just a very brief summary of the most important parts for the continuum problem.

1. Monads<sup>36</sup> are simple substances and they are entering compounds (#1) [qu'une substance simple, qui entre dans les composés] and because they are simple they have no parts, neither extension nor form nor shape nor divisibility (#3). So what we have here is not only a definition of a monad but these are the basic characteristics of the *principle* of the continuum.

On the other hand monads are well-determined and different (qualitatively discrete) from each other (#8-9) which is the *principle* of discontinuity.

So we have already two sides of the monad themselves - indivisibility and discreteness. This differentiates them both from (**P**) and (**I**) realms.

2. Further on (#10) Leibniz writes that every created being, and consequently the created monad, is subject to change, and further that this change is *continuous* in each [et même que ce changement est *continuel* dans chacune]. And the source of this continuity is monad's internal principle [d'un principe interne] (#11) which he will call "l'appetit" (#15). This is the first side of the continuum.

And the second is "la perception" (#14) which involve and represents a multiplicity in the unit (#13) – which is exactly what we were looking for in *syncategorematic continuum*.

And again we have two sides – we have the principle of continuous change and we have the principle of multiplicity in unity<sup>37</sup>. Both sides work together, because Leibniz realized that every natural change takes place gradually, something changes and something remains unchanged (#13) – that's the real secret of continuity.

The appetition is the principle of change from one perception (multiplicity in unity) to another (multiplicity in unity) (#15). So we have the variety of all particular changes existing only eminently (#38) (as a source) in the primary unity [l'unité primitive] (#47). And in this original simple substance [la substance simple originaire] there is a continuous force  $(#48)^{38}$ ...

Philosophy," in *The Cambridge Companion to Leibniz*, ed. Nicholas Jolley (Cambridge: Cambridge University Press, 1995), 293-98.

<sup>&</sup>lt;sup>35</sup> Richard T. W. Arthur, "Cohesion, Division and Harmony: Physical Aspects of Leibniz's Continuum Problem (1671-1686)," *Perspectives on Science* 6, no. 1-2 [Leibniz and the Sciences] (1998): 110.

<sup>&</sup>lt;sup>36</sup> I will use mainly Jonathan Bennett's and Robert Latta's translations.

<sup>&</sup>lt;sup>37</sup> In this systematical interpretation appetition will produce in phenomenal realm all timerelations and the perception will produce all spatial relations. So we can abstract them further in ideal realm as Time and Space.

Leibniz concluded in *Nature Itself* (1698): "not only is everything that acts an individual substance, but also every individual substance acts continuously...", translation by Jonathan Bennett, *Early Modern Texts* (2004 [cited 29 Feb 2008]); available from

# At the end we can say that the monad is foundation of the syncategorematic continuum in its reality and principle. It *neutralizes* it, being simple continuum (monad); and *preserves* it, being more than one (monads). Monad *vs.* monads, or better to say monad *and* monads. It is the only way phenomenal infinity can be sustained. And so the Law of Continuity.

## *The science of dynamics*

So it seems that the force is somehow very important for this interpretation – it unites and makes things dynamically continuous. There are many articles and analysis on the notion of Leibniz's force, so I will not make even a summary here, but I will pick up just some very basic and important facts about the formation of the concept<sup>39</sup>. Of course I will not talk about force only in a strict physical sense<sup>40</sup>, but nothing in my thesis violates recent scholarship conclusions on Leibniz's scientific thought.

**A**. In *De usu Geometriae* (1676) he wrote: "Only Geometry can provide a thread for the Labyrinth of the Composition of the Continuum, of maximum and minimum, and the unassignable and the infinite, and no one will arrive at a truly solid metaphysics who has not passed through that labyrinth."<sup>41</sup>. It is true that formulating the problem of continuum goes through analysis of infinity but the solution of the mature Leibniz is not only within the domain of the geometry. He gave up explaining everything with size, shape and motion (where geometry is quite strong) and decided to introduce the notion of force together with a whole new science – dynamics, which "treats force and the metaphysical entities"<sup>42</sup>. Furthermore in "Motion is not something absolute" (1686) he states: "And indeed *each substance is a kind of force of acting*, or an endeavor to

http://www.earlymoderntexts.com. After 7 years he wrote to Electress Sophie (31 Oct, 1705) about "entelechies or primitive forces [...] are the source of everything" – Leibniz, "Gp," VII, 565, translation by Lloyd Strickland, *Leibniz Translations* (Feb 2008 [cited 29 Feb 2008]); available from http://www.leibniz-translations.com. And finally in *Principles of Nature and Grace, Based on Reason* (1714) he said: "a substance is a being that is capable of action", translation by Bennett, *Early Modern Texts*.

<sup>&</sup>lt;sup>39</sup> Leibniz's development of this notion is a slow and many-faced. Very good introduction – where interactions between mechanics, scholasticism and dynamics are differentiated – is Garber, "Leibniz: Physics and Philosophy.". For a different short analysis of force's historical development, see François Duchesneau, "Leibniz's Theoretical Shift in the Phoranomus and Dynamica De Potentia," *Perspectives on Science* 6, no. 1-2 (1998).

<sup>&</sup>lt;sup>40</sup> Force is highly technical term which governs various laws, as Duchesneau writes: "Force is presented as a theoretical concept exceeding the intelligibility of geometrical concepts. And this new concept is presumed to own considerable regulative power for unifying the various empirical laws" Duchesneau, "Leibniz's Theoretical Shift in the Phoranomus and Dynamica De Potentia," 81. But on the other hand it is a pure metaphysical notion and as such it can be interpreted as a form, as did Leibniz.

 <sup>&</sup>lt;sup>41</sup> Leibniz, The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-1686 xxiii.

<sup>&</sup>lt;sup>42</sup> Garber, "Leibniz: Physics and Philosophy," 284. For example in "Discourse on Metaphysics" (1686), §18 he says: "Now, this force is something different from size, shape, and motion, and this shows us that - contrary to what our moderns have talked themselves into believing - not everything that we can conceive in bodies is a matter of extension and its modifications", translation by Bennett, *Early Modern Texts*.

change itself with respect to all the others according to certain laws of its own nature" [*italics added*]<sup>43</sup>, so we have to introduce a science which will combine forces, substantial forms and their measurement. I think that there is a connection between abandoning the early categorematic conceptions of the continuum (before 1676) and abandoning geometry as the only key solution for the labyrinth of the continuum.

**B**. In *Lettre sur la question, si l'essence du corps consiste dans l'etendue* (18 June, 1691) published by Leibniz in the Journal des Savant he says: "All of this shows that there is in matter *something else than the purely Geometrical*, that is, than just extension and bare change. And in considering the matter closely, we perceive that we must add to them some higher or *metaphysical* notion, namely, that of *substance, action* and *force*; and these notions imply that anything which is acted on must *act reciprocally*, and anything which acts must receive some reaction; consequently, a body at rest should not be carried off by another body in motion without changing something of the direction and speed of the acting body" [*added italics*]<sup>44</sup>. Here we can see not only the notion of force to be introduced but already the differentiation of the forces themselves – active (act on) and passive (act reciprocally). We can say that the drive – active force – is analogous to monad's appetite (that which is principle of change) and resistance – passive force – is analogous to monad's perception (that which unites multiplicity)<sup>45</sup>.

**C**. And 3 years later in his article *On the Correction of Metaphysics and the Concept of Substance* (published in *Acta Eruditorum*, March 1694) Leibniz mentions for the first time in print the notion *dynamica* as a new science on force: "...the concept of *forces* or powers, (which the Germans call *Kraft* and the French *la force*), and for whose explanation I have set up a distinct science of *dynamics*, brings the strongest light to bear upon our understanding of the true *concept of substance*<sup>\*46</sup>.

**D**. Finally in 1695 was published *Specimen Dynamicum* (in *Acta Eruditorum*) – which presented the metaphysical foundations of the dynamics and the foundations of posthumously published *Dynamica de potentia et legibus naturae corporae*. In

<sup>&</sup>lt;sup>43</sup> Leibniz, "Ag," VI-4, 1638; translation from Leibniz, The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-1686 333.

<sup>&</sup>lt;sup>44</sup> Or we can see another definition of dynamics: "I judged that it was worth the trouble to muster the force of my reasonings through demonstrations of the greatest evidence, so that, little by little, I might lay the foundations for the *true elements of the new science of power and action*, which one might call dynamics." – Duchesneau, "Leibniz's Theoretical Shift in the Phoranomus and Dynamica De Potentia," 84.

<sup>&</sup>lt;sup>45</sup> Interpretation on letter to De Volder – Leibniz, "Gp," II, 170. It can be compared with this excerpt from "Principles of Nature and Grace, Based on Reason" (1714): "The *qualities* of a monad must be its perceptions; a perception is a representation in something simple of something else that is composite. And a monad's *actions* must be its appetitions, which are its tendencies to go from being in one state to being in another, i.e. to move from one perception to another; these tendencies are the sources of all the changes it undergoes" [*italics added*] – Bennett, *Early Modern Texts*.

<sup>&</sup>lt;sup>46</sup> "...notionem virium seu virtutis (quam Germani vocant Krafft Galli la force) cui ego explicandæ peculiarem Dynamices scientiam destinavi, plurimum lucis afferre ad veram notionem substantiæ intelligendam" "De primæ philosophiæ emendatione et notione substantiæ" – Leibniz, "Gp," IV-469.

Specimen Dynamicum Leibniz presented systematic study on primitive force and its relation to the substantial forms; its priority to extension and that it is a definition of 'substance' (#2), its division of active, passive, primitive, derivative (#6-7), dead, live (#12), total, partial (#13) their measurement (#30) and so on... Further more he stated there: "In conformity with the Law of Continuity, which rules out jumps, rest can be considered as a special case of motion - that is, as vanishingly small or minimal motion - and equality can be considered as a case of vanishingly small inequality" <sup>47</sup>. Motion is the effect of force – so there is continuity in effects because there is *continuity in the force itself*. Effects are part of the phenomenal realm (**P**) and primitive forces are constituents of the real realm (**R**).

One more remark; it is well known that Leibniz incorporated Hobbesian *conatus* (endeavour) in his terminology<sup>48</sup> and embraced dynamics as after-Cartesian solution to some Cartesian problems (as conservation of quantity of motion). So we can schematize the appearance of force as: *there is no mechanical solution to the continuum problem, so we need a dynamical one.* 

#### (R)-(P)-(I) and the Law of Continuity

Let me make one final stroke by interconnecting the three realms: space and time are abstractions from the phenomenal world. But what has this phenomenal world in itself *that can be abstracted in a form of continuum* in the ideal realm? It has monadic substratum, it has substratic unities, so from phenomenal realm we abstract its real characteristics to make an ideal realm. In a letter to Arnauld (April 30th 1687) Leibniz postulated one crucial axiomatic statement which is "[an] identical proposition which varies only in emphasis: that what is not truly *one* entity is not truly one *entity* either<sup>\*\*49</sup>. Oneness (unity) is our final step and it is clear why Leibniz decided to use exactly the word "monad" as a constituent of the (**R**)-realm.

Let me make an overall summary of the continuum in his Weltanschauung.

<sup>&</sup>lt;sup>47</sup> Translation by Bennett, *Early Modern Texts*.

For more details, see Howard Bernstein, "*Conatus*, Hobbes, and the Young Leibniz," *Studies in History and Philosophy of Science* 11, no. 1 (1980) and Alan Gabbey, "Force and Inertia in Seventeenth-Century Dynamics," *Studies in History and Philosophy of Science* 2, no. 1 (1971).
 Cettining Leibnin and Armardd. *The Leibnin Armardd Communications* ed. B. C. Sleich, In.

<sup>&</sup>lt;sup>9</sup> Gottfried Leibniz and Arnauld, *The Leibniz-Arnauld Correspondence*, ed. R. C. Sleigh, Jr, trans. H. T. Mason, *The Philosophy of Leibniz - Fourteen of the Most Important Books on Leibniz's Philosophy Reprinted in Fifteen Volumes* (New York/London: Garland Publishing, 1985) 121. "... ce qui n'est pas véritablement *un* estre, n'est pas nan plus véritablement un estre." – Leibniz, "Gp," II, 97.

## $m K\alpha\gamma \Omega$ 2008/10

Realm	(R)	(P)	(I)
Ktalli	Real	Phenomenal	Ideal
Knowledge	Metaphysical	Physical	Mathematical
Constituents	Monads	Bodies/Aggregates	Figures <sup>50</sup>
Characteristics <sup>51</sup>	Perception and Appetite	Matter and Motion	Space and Time
Type of continuity	Dynamical	Syncategorematical	Pure abstract
Law of continuity	There is a force.	There's always a middle term $52^{2}$ .	There's a whole.
Features	Unity of contiguity (uniqueness) and continuity (change).	Contiguity to infinity (dense) <sup>53</sup> .	Continuity to infinity.
Number of boundaries between things	2 – (monads are different); 0 – (monad is undividable) <sup>54</sup>	2 - each thing has its own boundary <sup>55</sup> .	0 – interiorly unbounded.
Domain	Sufficient reason (unity of actual and possible).	Actual.	Possible.
Priority	Monad is prior to both part and the whole.	Part is prior to the whole.	Whole is prior to the part.
Simplest	One is simpler.	Part is simpler.	Whole is simpler.
Infinity: Compendia loquendi	Mirror	Fold	Infinitesimal
[Visual metaphor]	[.]	[]	[O]
Key	One	Part	Whole

Now we can demonstrate the *unity of the Law of Continuity*. It seems that we have three different types of continuum ( $\mathbf{R}$ )-( $\mathbf{P}$ )-( $\mathbf{I}$ ), but actually it is only one: real,

<sup>&</sup>lt;sup>50</sup> Figures, but maybe not numbers; see Crockett, "Continuity in Leibniz's Mature Metaphysics," 134. and compare it with Levey, "Matter and Two Concepts of Continuity in Leibniz," 87.

<sup>&</sup>lt;sup>51</sup> These can be regrouped in two analogous chains: Perception-Matter-Space and Appetite-Motion-Time. For example in *Nature Itself* (1698), §11 Leibniz locates the notion of primary matter in passive force – Bennett, *Early Modern Texts*.

 <sup>&</sup>lt;sup>52</sup> When formulated, Leibniz's Law of Continuity was explained as: there is always something *in between* during any change from small to large (or vice versa).

<sup>&</sup>lt;sup>53</sup> This is Crockett's "structural continuity" – Crockett, "Continuity in Leibniz's Mature Metaphysics," 128. But, of course (though it resembles the ideal realm), there is no parallel to his "metaphysical continuity" because it is based on density which doesn't make sense applied to figures and numbers – Crockett, "Continuity in Leibniz's Mature Metaphysics," 130.

<sup>&</sup>lt;sup>54</sup> We cannot talk about real boundaries in the (**R**)-realm.

<sup>&</sup>lt;sup>55</sup> "For by the very fact that the parts are discontinuous, each will have its own separate boundaries [terminos]" – Leibniz, "Ag," VI-2, 435. So the number of boundaries in the whole world will be always even. As Levey wrote: "discontinuous things, by contrast with continuous ones, are those whose boundaries are two." – Levey, "Matter and Two Concepts of Continuity in Leibniz," 84.

phenomenal and ideal altogether – (**RPI**). It is a *one whole with parts*, multiplicity united by force. And I am not talking about the World itself, only about the law, (it would have been incorrect to state that the world has its own monad or soul)<sup>56</sup>. I am not claiming that there are no differences between the realms – exactly the opposite – because there are differences the Law of Continuity is recursive and the engine of this recursion is the force.

#### **IV.** Conclusion

"In actual bodies there is only discrete quantity, that is, a multitude of monads or of simple substances, though in any sensible aggregate or one corresponding to phenomena, this may be greater than any given number. But a continuous quantity is something ideal which pertains to possible and to actualities only insofar as they are *possible*. A continuum, that is, involves indeterminate parts, while on the other hand, there is nothing indefinite in actual things, in which every division is made that can be made. Actual things are compounded as is *number out of unities*, ideal things as is a *number out of fractions*; the parts are actually in the real whole but not in the ideal whole. But we confuse ideal with real substances when we seek for actual parts in the order of possibilities, and indeterminate parts in the aggregate of actual things, and so entangle ourselves in the labyrinth of the continuous and in contradictions that cannot be explained. Meanwhile the knowledge of the continuous, that is, of possibilities, contains *eternal truths* which are never violated by actual phenomena, since the difference is always less than any given assignable amount." [*italics added*]<sup>57</sup>.

This is a really good resume by Leibniz. It is obvious he is talking only about two types of realm (actualities vs. possible), although he is much more precise when further differentiating real from phenomenal. By this citation we can assume that monadic level has the same type of syncategorematic structure as the matter itself, but what I wanted to show is that within the realm of monads next to their "discreetness" lies the principle of continuous change and the principle of oneness (simple substances) – what is not truly *one* entity is not truly one *entity* either. This is the difference between

<sup>&</sup>lt;sup>56</sup> In *De mundo praeseti* (1684-1686) Leibniz wrote: "The aggregate of all bodies is called the world, which, if it is infinite, is not even one entity, any more than an infinite straight line or the greatest number are. So God cannot be understood as the World Soul: not the soul of a finite world because God himself is infinite, and not of an infinite world because an infinite body cannot be understood as one entity [unum Ens], but that which is not one in itself [unum per se] has no substantial form, and therefore no soul." – Leibniz, "Ag," VI-4, 1509. More about this in Gregory Brown, "Leibniz's Mathematical Argument against a Soul of the World," *British Journal for the History of Philosophy* 13, no. 3 (2005). But this can be compared with his earlier thoughts in *On the Secrets of the Sublime* (1676): "[God] exist as a whole soul in the whole body of the world" – Leibniz, "Ag," VI-3, 474; translation from Leibniz, *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-1686* 48.

<sup>&</sup>lt;sup>57</sup> Leibniz in a letter to De Volder (19 Jan, 1706) – Brown, "Leibniz's Mathematical Argument against a Soul of the World," 468; Leibniz, "Gp," II, 282. My claim is stronger than what we see here in the last sentence. It's not only about this epistemologically unassignable error, but it is because there is multiplicity in unity and unity of the multiplicity.

syncategorematical and dynamical continuum which I tried to present – introduction of the force. As Leibniz said "The cohesiveness of bodies is the *quantity of force* needed to destroy their contiguity" [*italics added*]<sup>58</sup>.

The same force which keeps together the unity of the world is the force to grasp the multiplicity in unity (in our mind and in reality) and it is the force which sustains syncategorematic continuum by making it one. And the same force, *abstracted*, can produce the ideal Space (by monads' perception) and ideal Time (by monads' appetite)<sup>59</sup>; and in the opposite direction to *reduce* them via phenomenal world to the monads themselves. And the same force makes this world dynamical and thinkable. So the solution of the continuum problem cannot be plainly mathematical (ideal), and it cannot be plainly physical (phenomenal), but it can be both... and united by one (real) metaphysical principle. And even the historical period – Baroque – is always syncategorematically folded, as Deleuze said: "the characteristic of the Baroque is the fold that goes on to infinity"<sup>60</sup>. But if the fold should be one it needs a deeper dynamical level. So in Leibnizian world each doubling in itself is dynamical, syncategorematically and geometrically ideal.

We can make another (this time epistemological) parallel; Leibniz wrote in *Contingency* (1686) that every analysis of a contingent proposition continues to infinity – you have a cause for the cause for the cause for the cause... so you will never have a complete demonstration. But on the other hand there is always an underlying complete and final reason for the truth of the proposition (only God completely grasps it, as being the only one who can whip through the infinite series in one stroke of the mind)<sup>61</sup>. And so contingent cause is contiguously continuous and sufficient reason is dynamically continuous.

So at the end we should have only one Law of Continuity which corresponds to the three different realm structures. Which means that continuity will have 3 different structural manifestations but the law is one and the same – so even here we have multiplicity in one unity? The Law itself is meaningful only applied in (**RPI**) together and this is a subtle hint against the recent scholarship debate about "Was Leibniz an

<sup>&</sup>lt;sup>58</sup> Leibniz, "Ag," VI-3, 94; translation from Leibniz, *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-1686* 19. Sometimes the influence of Descartes is still visible when Leibniz is talking not about force, but about motion as in *On the Secrets of the Sublime* (1676), but the idea is similar: "Matter is a discrete being [ens discretum], not a continuous one; it is only contiguous, and is *united* by motion or *by a mind* of some sort." [*italics added*] – Leibniz, "Ag," VI-3, 474; translation from Leibniz, *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-1686* 47.

<sup>&</sup>lt;sup>59</sup> Compare this with the hint given by Garber: "extension, is properly speaking, a direct consequence of the properties bodies have by virtue of which they resist penetration by other bodies" – Garber, "Leibniz: Physics and Philosophy," 291. But than Garber is puzzled by the diagram from 1715 in the letter to De Bosses where primitive forces are only in this part of the (**R**)-realm which is *substantia composita* – Leibniz, "Gp," II, 506. And I am not sure why Garber expects them "on the other side of the chart, in the characterization of semisubstances"... – Garber, "Leibniz: Physics and Philosophy," 298.

<sup>&</sup>lt;sup>60</sup> Gilles Deleuze, "The Fold," Yale French Studies 80 (Baroque Topographies: Literature/History/Philosophy) (1991): 227.

<sup>&</sup>lt;sup>61</sup> Bennett, Early Modern Texts.

📠 Καγρι	2008/10
---------	---------

idealist?", because what is most important in Leibniz dynamics about continuity... is the reciprocity (**RPI**) of force, the reciprocity (**RPI**) of unity-multiplicity pair which is a pair connected by force<sup>62</sup>.

#### Leibniz and Zeno – the last words

Of course, we cannot deal with the continuum problem non-mentioning Zeno, though there was no time, nor space, nor motion to include him in this paper. Still I would like to add one short question and even shorter answer. I was thinking can we illustrate the Leibniz solution to the continuum problem by re-reading for example *Dichotomy* as a dialogue and giving it a possible Leibnizian answer.

**Zeno**: – That which is in locomotion must arrive at the half-way stage (1/2) before it arrives at the goal (1). And than if you pick the half-way stage (1/2) as your new goal you must first arrive at its half-way stage (1/4). And so on, 1/8, 1/16, 1/32... to infinity. So how is it possible, dear Leibniz, to overcome that infinity?

Leibniz: – By force, Zeno, by force...

#### BIBLIOGRAPHY

ARTHUR, Richard T. W. (1998) "Cohesion, Division and Harmony: Physical Aspects of Leibniz's Continuum Problem (1671-1686)." *Perspectives on Science* 6, no. 1-2 [Leibniz and the Sciences] 110-35.

ARTHUR, Richard T. W. ""A Complete Denial of the Continuous"? Leibniz's Law of Continuity." In *Synthese*, 36, forthcoming.

ARTHUR, Richard T. W. "From Actuals to Fictions: Four Phases in Leibniz's Early Thought on Infinitesimals." In *Studia Leibnitiana*, 32, forthcoming.

BENNETT, Jonathan. (2004) *Early Modern Texts* [cited 29 Feb 2008]. Available from http://www.earlymoderntexts.com.

BERNSTEIN, Howard. (1980) "Conatus, Hobbes, and the Young Leibniz." Studies in History and Philosophy of Science 11, no. 1: 25-37.

BROWN, Gregory. (2005) "Leibniz's Mathematical Argument against a Soul of the World." *British Journal for the History of Philosophy* 13, no. 3 : 449-88.

BROWN, Stuart. (1995) "The Seventeenth-Century Intellectual Background." In *The Cambridge Companion to Leibniz*, edited by Nicholas Jolley, 43-67. Cambridge: Cambridge University Press.

CROCKETT, Timothy. (1999) "Continuity in Leibniz's Mature Metaphysics." *Philosophical Studies* 94, no. 1-2 119-38.

DELEUZE, Gilles. "The Fold." *Yale French Studies* 80 (Baroque Topographies: Literature/History/Philosophy) 227-47.

DUCHESNEAU, François. (1998) "Leibniz's Theoretical Shift in the Phoranomus and Dynamica De Potentia." *Perspectives on Science* 6, no. 1-2 : 77-109.

<sup>&</sup>lt;sup>62</sup> I can't resist abbreviating "reciprocity" as RPI: (a) concerning etymology of "reciprocal" (recus + -procus + -ity) and (b) concerning the systematicity of Leibnizian Weltanschauung.

# <u>Μικαγθι</u> 2008/10

GABBEY, Alan. (1971) "Force and Inertia in Seventeenth-Century Dynamics." *Studies in History and Philosophy of Science* 2, no. 1 : 1-67.

GARBER, Daniel. (1995) "Leibniz: Physics and Philosophy." In *The Cambridge Companion to Leibniz*, edited by Nicholas Jolley, 270-352. Cambridge: Cambridge University Press.

HARTZ, Glenn A., and J. A. Cover. (1988) "Space and Time in the Leibnizian Metaphysic." *Noûs* 22, no. 4, 493-519.

LEIBNIZ, Gottfried. (1965) "Die Philosophischen Schriften Von Gottfried Wilhelm Leibniz." In *OLMS Paperbacks*. Hildesheim: Georg Olms Verlag.

LEIBNIZ, Gottfried. (1923) "Sämtliche Schriften Und Briefe." Darmstadt/Leipzig: Otto Reichl Verlag.

LEIBNIZ, Gottfried. (2001) *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-1686.* Translated by Richard T. W. Arthur. Edited by Daniel Garber and Robert C. Sleigh, *The Yale Leibniz.* Yale: Yale University Press.

LEIBNIZ, Gottfried, and ARNAULD (1985) The Leibniz-Arnauld Correspondence. Translated by H. T. Mason. Edited by R. C. Sleigh, Jr, The Philosophy of Leibniz - Fourteen of the Most Important Books on Leibniz's Philosophy Reprinted in Fifteen Volumes. New York/London: Garland Publishing.

LEVEY, Samuel. (1999) "Matter and Two Concepts of Continuity in Leibniz." *Philosophical Studies* 94, no. 1-2, 81-118.

RESCHER, Nicholas, (1979) *Leibniz: An Introduction to His Philosophy*. Totowa: Rowman and Littlefield.

RUSSELL, Bertrand (1900) A Critical Exposition of the Philosophy of Leibniz, with an Appendix of Leading Passages. Cambridge/New York: Cambridge University Press/Macmillan.

STRICKLAND, Lloyd. *Leibniz Translations* (2008) [cited 29 Feb 2008]. Available from http://www.leibniz-translations.com.