

# DU CHÂTELET’S PHILOSOPHY OF MATHEMATICS

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## ABSTRACT

Du Châtelet’s main works articulate a stance on the nature of mathematical objects, our knowledge of mathematical truths, and how such truths apply to the physical world. I begin by outlining her ontology of mathematical objects. She is an idealist, and mathematical objects are fictions dependent on acts of abstraction. Next, I consider how this idealism can be reconciled with her endorsement of necessary truths in mathematics. Finally, I discuss how mathematics and physics relate within Du Châtelet’s idealism: we can, she holds, be justified in drawing conclusions about physical objects from mathematical premises, even if these conclusions are only approximately true.

At the beginning of her intellectual career, in 1734, Émilie Du Châtelet wrote to Maupertuis that she wished “to become a mathematician [*géomètre*]” (2018, I:133; all translations mine). She went on to work largely on mathematical physics and its philosophical foundations. Her lengthiest theoretical works—the *Institutions de physique* and her commentary on Newton’s *Principia*—articulate a stance on the nature of mathematical objects, our knowledge of mathematical truths, and how such truths apply to the physical world.<sup>1</sup> But she mainly aims to clarify applications of mathematics in natural philosophy, rather than to give a freestanding account of pure mathematics.

I advance interpretations of three aspects of Du Châtelet’s views on mathematics, in the context of these broader views: her idealism about mathematical objects (section 1); the necessity of many mathematical truths (section 2); and her defense of inferences between mathematics and physics in applied mathematics (section 3). I also set these views in the context of Du Châtelet’s broader philosophical project, especially her idealist metaphysics and goal of giving philosophical foundations for physics.

## I. Du Châtelet’s Idealism about Mathematical Objects

To start with Du Châtelet’s general metaphysical picture: matter is not fundamental in the created order. There are more fundamental, immaterial simple substances. Yet we cannot observe simple substances directly, and physics deals merely with non-fundamental bodies.

Interpreters disagree on precisely how matter is grounded in simple substances, and in particular on whether matter is mind-dependent. On Marius Stan's reading, the existence of matter is solely grounded in simple substances that are not mind-like, so it "does not require any [non-divine] mental facts to ground it" (2018, 493). Caspar Jacobs, by contrast, argues that matter's existence is partly grounded in mental facts. At least some immaterial simple substances, though fundamental, seem to be *souls* and hence mind-like. On this reading, then, mental facts are a necessary but not sufficient condition for the existence of matter (Jacobs 2020, 71; see also Carson 2004, 169). The position I defend here, which focuses on mathematical objects, is compatible with either of these readings.

Mathematical objects are mind-dependent in a way that material things are not, in that they are abstracted from material things.

Space is to real beings, as numbers are to things numbered, which things come to be similar, and each form a unity with respect to Number, because one abstracts from the internal determinations of these [numbered] things [*fait abstraction des determinations internes de ces choses*], and considers them only insofar as they can make up a multitude...without a multitude of things that are counted, there would be no real and existent Numbers. (1740, 107)

She compares the ontological status of numbers to that of space and geometrical extension, which are "ideal" beings (1742, 104; 112). Extension, like number, is "formed by abstraction" from things, but it "is not" the things from which it is abstracted (107). In other words, extension is not numerically identical to concrete things. Nevertheless, Du Châtelet writes that

All the sciences, and most of all mathematics, are full of...fictions, which are one of the greatest secrets of the art of invention, and one of the greatest resources for achieving solutions to the most difficult problems, which the understanding on its own can only rarely attain; thus, one must allow a place for these imaginary notions whenever they can be substituted for real notions without prejudice to the truth, as when one makes use of the system of Ptolemy to obtain solutions to many problems of astronomy. (1742, III-12)

Mathematics can be used to derive true results, such as astronomical predictions, even though many or even all mathematical objects are "fictions." These claims apply both to continuous magnitudes treated in geometry and to numbers, which were traditionally understood as discrete magnitudes.

How then do mathematical objects and concrete things relate? The passages quoted above suggest that mathematical objects depend both on actual material things and on acts of

abstraction. A number, for example, “actually exists only insofar as there exist things [*choses*] which one can reduce as units under the same category [*réduire comme des unités sous la même classe*]” (1740, 120; see also 12). Mathematical objects can thus be seen as partly metaphysically grounded in material things, though also partly grounded in minds like ours. In turn, material things have their grounds in fundamental simple substances, including simple substances that are not mind-like. Transitively, mathematical objects can be partly grounded in simple substances that are not mind-like. In the next section, we’ll see that the divine intellect also plays a role in this grounding story.

A complication is Du Châtelet’s distinction between magnitudes and mathematical objects. The manuscript of the *Institutions* describes magnitude or size as an “internal” property of a “thing” (*chose*) (Du Châtelet 1738–40, ff. 33<sup>r</sup>–33<sup>v</sup>). Magnitudes in this sense, then, are properties of material things. For example, she holds that each particle of matter has a size (f. 32<sup>v</sup>).

This might suggest a broadly Aristotelian reading on which mathematical objects are identical to magnitudes that are in material things. Abstraction would consist in merely disregarding non-quantitative properties of things, which “one has neglected in forming the idea” of a mathematical object (1740, 107). In that case, mathematical objects would be no more ideal than the token properties of concrete material things.

But such an Aristotelian reading seems incorrect. Du Châtelet denies that a mathematical object can be identical to a real material thing or its properties, and therefore mathematical objects cannot be identical to the magnitudes in material things, since these are just properties of material things (1742, 112). Her reasons for denying this largely follow from the fact that mathematical objects are “ideal” entities, or “fictions,” which I further discuss in section 3 below (1740, 104; 106).

First, some properties of mathematical objects are also ideal and “abstract,” unlike the properties of matter (1742, 111). Token properties of bodies, by contrast, are “concrete” (1740, 106). The size of a particle, understood as a physical magnitude, is located somewhere and somewhen, but a number is not. For example, geometrical lines have no width, and do not exist in material nature; a similar point applies for geometrical surfaces (105; 189). Or to take another example, we can assume that a geometrical solid is homogeneous and has exact boundaries, whereas matter is actually an irreducibly heterogeneous mixture that lacks exact boundaries (191–92; 200–201; Leibniz 2001, 314).

Second, mathematical and material entities have different identity conditions:

When one divides a [geometrical] line in whatever way, and into however many parts, as one wishes, the same line always results from reassembling the parts, however one transposes these parts among themselves: it is the same for surfaces [i.e., planes] and for geometrical bodies [i.e., solids]. (1740, 99; see also 1742, 190)

So geometrical objects preserve their identity and properties across certain rearrangements, or decompositions and reaggregations, of their parts. In particular, these are operations that preserve the external limits of the geometrical object in question (as opposed to, for example, reassembling the parts of a given line segment into multiple shorter segments). But we cannot assume that the same body always results from reassembling its parts in a different order. The result may be a body of a different natural kind, with a different density and volume (1742, 216–17). A related contrast is that geometrical objects are taken to have their properties independent of their surroundings: Euclidean geometry assumes that figures can be superimposed without deformation. The shape of a body, however, partly depends on causal interaction with its surroundings (207). Admittedly, these contrasts don't seem as sharp as Du Châtelet presents them. For example, a body's mass will be invariant across rearrangement and decomposition. If the magnitude of a body is construed as its mass, then both bodies and geometrical objects can preserve their magnitude across certain rearrangements and decompositions. Still, she seems justified in concluding that bodies are “determined” by their parts and surroundings in ways that geometrical objects are not (112; Reichenberger 2021, 350).

Third, natural numbers and fundamental simple substances are essentially unified wholes (Du Châtelet 1742, 70). On her view, however, matter is essentially not a unified whole, since it is indefinitely divisible. So there is a property that numbers essentially have, but material things essentially lack.<sup>2</sup>

In sum, Du Châtelet holds that some properties of mathematical objects can't be identical to the properties of material objects, even if mathematical objects are partly grounded in physical magnitudes. Mental activities such as abstraction are supposed to explain how mathematical objects wind up with these properties. She says less than might be hoped about the nature of abstraction, but evidently, it does not just mentally subtract or leave aside properties that are already in material beings. Abstraction is a creative “power” of “forming...imaginary beings that contain just the determinations that we want to examine” (1742, 105; III).

Although the representations formed by abstraction are imaginary, it does not follow that their representational content (“the determinations that we want to examine”) is also

merely imaginary. The understanding is also involved in abstraction. Some geometrical figures can be conceived through the understanding, but not imagined (158–59). An example might be the solid that Torricelli showed to have infinite surface area but finite volume. Algebra, which is based “only” in the “understanding,” assists in these cases (3). The unity of a function, she holds, is grasped by the understanding (III; see also 33–34; 249; 410).

As it stands, Du Châtelet’s conception of mathematical objects is vulnerable to one objection from Platonists. If mental activities such as abstraction play such a crucial role in generating mathematical content, the objection goes, it seems that mathematics becomes unacceptably subjective and arbitrary. Why assume that our powers of abstraction track the truth? Du Châtelet invokes the principle of non-contradiction as a constraint. But this cannot provide an adequate response, as she acknowledges that mathematics does not get most of its content from the principle of contradiction. Her account of mathematical necessity, as I detail in the next section, introduces further constraints on the content of mathematics. These constraints reduce the force of the mathematical realist’s objection. But they also bring Du Châtelet’s account closer to Platonism than it first appears.

## 2. Necessary Truths in Mathematics

Du Châtelet holds that some mathematical truths are necessary. Our acts of abstraction appear to be contingent, so the necessity of mathematical truth evidently requires another source. She assigns the divine intellect an important role in fixing mathematical truths. But her views interestingly diverge from those of earlier rationalists, such as Leibniz.

A passage early in the *Institutions* states that geometrical truths are necessary, while also potentially suggesting a kind of logicism about geometry:

In geometry where all truths are necessary, one only makes use of the principle of contradiction. For in a triangle, for example, the sum of the angles is determinable only in a single manner, and the angles absolutely must equal the sum of two right angles. (Du Châtelet 1740, 24–25).

I want to begin by showing that, in context, her claim that “only” the principle of contradiction is used in geometry is not as strong as it might seem.

To be sure, Du Châtelet takes logical demonstration to be crucial in geometry and science more generally (1740, 17). It is a necessary condition for mathematical truth. If it did not hold,

There would no longer be any truth, even in numbers, and each thing could be or not be, according to the fantasy of each, so that two and two could make four just as much as six, or even both at once (18–19; see also 1738–40, 29<sup>v</sup>–30<sup>r</sup>)

Like Leibniz (1875–90, IV:363), she rejects a Cartesian account on which intuition of eternal truths and a “lively, internal sentiment of clarity and evidence” are the basis for mathematical reasoning (Du Châtelet 1740, 16–17).<sup>3</sup> While Du Châtelet grants that we appear to have clear and distinct ideas of mathematical objects such as equilateral triangles, she holds that these appearances may mislead us. We should rely instead on rigorous deductive proofs (17).

Nevertheless, she does not construe geometrical proof as resting on logic alone. While her views here are not unusual in an eighteenth-century context, it will nevertheless be worth clarifying them. First, her discussions of geometrical examples make clear that the logical necessity she has in mind is hypothetical. Geometrical proof relies on logical consequence relations, but it must begin with irreducible, non-logical facts. One of her examples is the problem of finding the unknown length  $L$  of one side of a trapezoid. If we are given the values for the three other side lengths and the two angles opposite to  $L$ , then  $L$ 's length will be “determined by these givens [*données*]” (1740, 61). The length “follows from” them with hypothetical necessity (1740, 61).<sup>4</sup> But the givens themselves are contingent rather than logically necessary:

These givens do not in the least have intrinsic determinations, which determine them to be together, and their magnitude [*grandeur*] can vary and be such as he who gives the problem decides. (1740, 61; see also 17–18)

In other words, it is possible for there to be trapezoids with the same angles and different side lengths, or vice versa. At least some of the givens in a geometrical problem, this passage suggests, are not logically necessary.

Second, an essential part of a Euclidean geometrical demonstration is to show “how things must be done in order to construct” the relevant geometrical objects (Du Châtelet 1740, 17–18). The demonstration necessarily this constructive element, which can be realized by diagrams.

Third, Du Châtelet does not take basic operations on numbers to be purely logical. On her view, the number 1 is the unit or “a non-composite number” on which all other natural numbers depend (1740, 133). However, this is not logical dependency. Instead, the number 1, as a unit, gives the “sufficient reason” for higher numbers, presumably in conjunction with addition or counting operations (133). For Du Châtelet, the principle of sufficient reason is

non-logical and irreducible to the principle of contradiction (22). Therefore, the basic laws of arithmetic are not logical.

Even with these clarifications in hand, the passage we began with seems in tension with Du Châtelet's idealism about mathematical objects. The passage, though not committing her to a logicist account of geometry, does state that "all" geometrical truths are necessary. Its context is a discussion of two fundamental principles of our knowledge and reasoning: the principle of contradiction and the principle of sufficient reason. The principle of contradiction is the only one needed for necessary truths, but for contingent truths, the principle of sufficient reason is needed as well (Du Châtelet 1742, 22–23). So in saying that in geometrical proofs, "only the principle of contradiction is used," Du Châtelet is emphasizing that geometry only makes use of the principle of necessary truths, that is, the principle of contradiction.

Given her idealism about mathematical objects, however, the objects of geometry are partly mind-dependent. They depend on acts of abstraction, carried out by minds like ours. But it doesn't seem necessary that such minds exist, let alone that they actually carry out the relevant acts of abstraction. For example, humans might have contingently failed to use the power for abstraction to develop geometry. If mathematical truths themselves depend only on contingent acts of abstraction, then it is hard to see how these truths are logically necessary.

One way to avoid this tension would be to read Du Châtelet as holding that all mathematical truths concern merely possible objects. Mathematical claims would then be made true by God, as the ground of what is possible. Du Châtelet thinks *possibilia* are "in God," and specifically in the divine understanding, which is "the eternal region of truths" that "contains everything that is possible" (1740, 46; 1742, 74). Truths about the essences of possible things are necessary truths, and would hold even if nothing was ever created (1740, 64–65). On this reading, she might argue as follows: all mathematical truths concern possible objects; truths about possible objects are *de dicto* necessary; therefore all mathematical truths are *de dicto* necessary.

Unfortunately for this reading, the necessary truths in question are not first-order mathematical truths. They are metaphysical truths about possibility and impossibility. For Du Châtelet, all such truths about possibility are necessary. To use her own example:

(a) *Necessarily, it is possible that Alexander the Great did not invade Persia* (1740, 43).

Even in possible worlds where Alexander does not exist, it is true that he might have existed and not invaded Persia. This fact is grounded in the divine understanding, so it is independent of God's voluntary choice to create the actual world (46–50). It is necessary because God is a necessary being, and facts about the divine understanding are grounded in the divine essence. Now, Du Châtelet sees that this doctrine also applies to mathematical truths, noting for example that

(b) *Necessarily, it is possible that triangles have three sides* (60).

Whether or not there actually are any triangles, (b) is true in virtue of God's understanding. There is no doubt that Du Châtelet endorsed (b), but the interesting question is whether Du Châtelet thought all mathematical truths are necessary *simpliciter*, as in

(c) *Necessarily, triangles have three sides*.

Her general doctrine that, if it is possible that  $p$ , then necessarily it is possible that  $p$  does not, however, entail the necessity of mathematical truths like (c).

A second reading would take the necessary truth of propositions such as (c) to be solely based in facts about the divine intellect. Then, mathematical truths are true in all possible worlds, insofar as they are grounded in the divine intellect. By contrast, the laws of nature, even if they hold necessarily given that the actual world was created, might not hold if God had chosen to create a different possible world. Such a reading would do justice to her claim that in geometry, all truths are necessary.

Reading Du Châtelet this way would bring her close to Leibniz. The truths of arithmetic and geometry are, for Leibniz, eternal truths grounded in God. Leibniz often depicts them as actual intentional objects of the divine understanding, though divine immensity and eternity may also play a grounding role (Leibniz 1875–90, II:49; II:305; V:210; VII:184; VII:275–78, 200I, 334; 20II, 123; 308–9; 337–38). These mathematical truths have “absolute” or “brute necessity” [*brutae necessitatis*] (IV:391). They depend on God, not on which world God chooses to create, and therefore are true in every possible world. Such a reading might be particularly appealing because for Du Châtelet, anything logically possible is, in a minimal sense, a *being*, grounded in the divine intellect (1742, 61; cf. Descartes 1964–76, VII:116). So all consistent mathematics could be grounded in the divine intellect.

Leibniz can leave room for abstraction as a way for us to epistemically access necessary mathematical truths. He sometimes suggests that mathematical truths, though necessary and grounded in the divine understanding, are accessed through contingent acts of abstraction, mediated by our internal senses (Leibniz 2011, 228; Reichenberger 2021, 342). Our abstracting abilities reliably track the mathematical truths grounded in the divine intellect, it seems, because God is the creating cause of our intellects. So Leibniz need not hold that our mathematical knowledge requires directly grasping truths in the divine intellect.

Yet at least in the case of numbers, Du Châtelet maintains a distinction between mathematical objects that are actual and those that are merely possible or conceivable. There are merely “possible numbers,” independent of actual created things, that are grounded in the divine intellect (107). Truths about possible numbers apparently hold in all possible worlds. To this extent she follows Leibniz. This affords a partial response to another objection to abstractionist approaches to mathematics often raised by Platonists, different from the one about arbitrariness, namely that there are many more mathematical objects than can be produced by finite minds and their activities of abstraction.

But possible numbers are only part of the story. Actual, “real and existent” numbers must be grounded in concrete, countable things, as well as in mental acts of abstraction (1740, 107). This gap between possible and actual mathematical objects seems incoherent if all mathematical objects are assumed to be strictly necessary. But if we look more carefully at the claim that all truths in geometry are necessary, she insists on is that there are essences, from which consequences necessarily follow. For example, if something is a triangle, then necessarily, its angles must be equal to the sum of two right angles (1742, 70; Jacobs 2020, 68n13). She makes a similar point for natural numbers (1742, 70–71). The necessity in question is *de re* rather than *de dicto*, and does not require that triangles and numbers exist necessarily. So her claims are consistent with propositions about triangles or natural numbers being false or vacuous if triangles or natural numbers do not actually exist. This reading fits well with her emphasis on the principle of contradiction in mathematics, which plays a role not just in proofs, but also in the identity conditions of essences.

To see why she thinks some mathematical propositions are contingent, we can begin with her claim that some mathematical objects are contingent. Actual mathematical objects depend on there being powers of abstraction. It is “hardly necessary” that minds like ours, which form these mathematical objects by abstraction, actually exist (1740, 65; 107). Therefore, it is not necessary that some mathematical objects exist. Indeed, it seems that some

mathematical objects are not actual at all times: large numbers may need to be generated by abstraction.

From the contingency of mathematical objects, it does not automatically follow that mathematical *propositions* are contingent. Mathematical objects and mathematical truths could differ in modal status. For example, there might be necessary conditional mathematical truths where the antecedents are contingently true, depending on which mathematical objects exist.

But Du Châtelet thinks the modal status of mathematical objects can indeed determine the modal status of mathematical truths. Some mathematical truths obtain in virtue of the actual existence of mathematical objects. She argues repeatedly that precisely because of the mind-dependent status of mathematical objects, some mathematical truths and demonstrations cannot be generalized to fully mind-independent substances (1742, 102–104; 113; 194). These arguments presuppose that if a mathematical object is partly dependent on finite minds, then at least some truths or demonstrations referring to that object will partly depend on finite minds. We've seen that she accepts the antecedent: some mathematical objects are contingent. She also accepts that if  $x$  is partly dependent on finite minds, then since finite minds are contingent,  $x$  is contingent too. It follows that she thinks some mathematical truths and demonstrations are contingent: there are possible worlds where they are false and unsound. This conclusion fits with her depiction of mathematics as fictional: some truths about objects in a fiction depend on the fiction. In worlds where the fiction doesn't exist, these truths wouldn't hold.

Which truths might these be? Du Châtelet says little about this. One example might be propositions in calculus. In some of her formulations, calculus appeals to fictional entities that are not even possible objects, in her view, such as “infinitely small straight lines” (1742, 265). Other examples are propositions in plane geometry, such as the Euclidean definition of ‘parallel line,’ that essentially take “extension” as “unlimited” or even “infinite” (109). Another case is more puzzling. Truths about ‘actual’ numbers apparently depend on the existence of multiple “things” and minds like ours that “count” them (107). If so, there is some sense in which arithmetic is true only in worlds with multiple things and the right kinds of beings to count them.

Now, this suggestion need not be understood as making mathematical truths dependent on the token activities of individual thinkers: it might be cashed out in terms of general facts about possible worlds. Another way to make her claim more palatable would be to limit it to some, but not all, truths about numbers. There is some textual support for this.

She stresses that actual natural numbers have a distinctively ordinal character linked to temporal succession (I2I). Time is dependent and finite minds, so it does not exist in all possible worlds. But truths about the *cardinal* properties of natural numbers might hold in all possible worlds, if they turn out to be logical truths.

Despite these caveats, one might worry that her account of ‘actual’ mathematical objects and truths runs together the conditions under which we acquire mathematical beliefs with the grounds of mathematical truth itself. Also, the very idea of a distinction between actual and possible may seem suspect in the case of mathematical objects.

To better understand her motivation for this doctrine, recall that the grounds in question, in the divine intellect, just give us all the *logical* possibilities. While it may be tempting to insist that this settles the key question for mathematics—namely, consistency—this does not automatically follow. For there are also more demanding senses of possibility, such as nomological possibility, or some notion of constructability or geometrical possibility. A mathematical proposition could be logically possible (that is, consistent) while also being impossible in certain worlds.<sup>5</sup> Du Châtelet’s chief concern is with applied mathematics. Truths about mere logical possibilities do not entail substantive truths about the actual world, or about those possible worlds containing matter and minds similar to ours. This may lead her to focus on more demanding senses of possibility. Her talk of establishing a truth about ‘actual’ and not just ‘possible’ numbers might then be glossed in terms of showing that a truth is possible in more demanding senses that are not entailed by logical possibility.

### 3. Mathematics, Physics, and Approximation

We can, in Du Châtelet’s view, use mathematics to effectively represent, and reason effectively about, the physical world. Here are some typical passages:

- 1) “The same thing happens in nature as in geometry,” at least with respect to sufficiently general and abstract properties (I742, 34).
- 2) An example of the previous thesis is that, universally, matter and change is “actual[ly]” continuous (I07; I27; 435–36).
- 3) “Geometry is the key to all doors” in physics or the investigation of nature (2018, I.500). The seventeenth-century scientific “revolution,” she holds, acquired “solid foundations” by combining “geometry” with observation and experiment (I740, 12; 5; 9).

- 4) These points also hold for Cartesian analytic geometry (3–4; 14), and for calculus as “the geometry of the infinite” (which Newton has provided with “all the certainty of that of the ancients” via his method of first and last ratios) (1759/1990, 9).
- 5) Newton’s demonstration of “the proportion in which gravity acts,” to consider a key example, can be directly attributed to his use of calculus or “profound geometry” (1759/1990, 7–8).
- 6) Newton’s use of mathematics here makes his theory a “demonstrated truth,” as opposed to Hooke’s hypothesis of universal gravity, which she regards as a lucky guess (1759/1990, 6–7; Smith 2022, 269–70). Newton, not Hooke, was properly “guided by geometry,” and the use of mathematics makes his theory a fecund source of further discoveries (1740, 6).

Why then does the same thing happen in nature and geometry? This is a pressing question because Du Châtelet thinks mathematical objects are useful, in part, as are products of abstraction that are not numerically identical to physical objects.

Mathematics, for Du Châtelet, is concerned only with “magnitude” (*grandeur*), and thus abstracts from all “internal difference” among the parts of its objects (1740, 119; 99). Nevertheless, it is permissible “to put the imaginary notion” of a geometrical object “in the place of the real one,” that is, the notion of a physical body (1740, 119). In fact, mathematical representations are cognitively unavoidable for us. Without them, we would need to represent more particulars than we can grasp through our “understanding alone” (106). An abstract mathematical representation of a physical thing provides a needed simplification that makes our inferences “intelligible” (1742, 250). This is one reason why she holds that mathematical fictions are used in “all” sciences (III). More broadly, abstract representations are supposed to allow mathematics to be rigorous and general.

This picture suggests, however, that insofar as they use mathematical representations, physical theories are only approximately true. As Du Châtelet puts it:

To reduce physical effects to mathematical calculations, we are always obliged to make a number of assumptions, and when we then wish to come back from mathematical calculations to physical effects, we find that there is a considerable loss [*bien du déchet*] of exactness and precision. (1740, 394).

Her claim that we are always obliged to make simplifying assumptions is not, I think, merely rhetorical. Because of the limitations of our faculties, “we can only see objects by parts, and to consider the composite, it is always necessary for us to simplify it” (384). That is, the actual world is too complex, given our limitations, to represent with precision. We must use

mathematical approximation: the ‘parts’ she refers to here are not the material parts of bodies, but rather simpler components of a mathematical model.

An example from natural philosophy she considers is the simplification required to treat the trajectory of a projectile as a parabola traced by a center of mass point. Air resistance is ignored, and the projectile is considered symmetrical and uniform in density (1742, 237; 319; 356). But this is a “fictional case” which is “by no means exactly true” of actual projectiles (1738–40, 342<sup>v</sup>; 1740, 395). Nevertheless, if the projectile is sufficiently heavy and compact, the results will be acceptable approximations for most purposes. Such approximation will not produce an “appreciable [*sensible*] error,” and divergences from truth can even be “counted entirely as nothing” (1742, 395; 396; see also 206; 332–33). While these points refer to contingent approximation in natural philosophy, they generalize, given that mathematical representations of the physical world “always” fall short of exactness (1740, 394).<sup>6</sup>

Her unusual translation of Newton’s fourth rule of reasoning in natural philosophy seems to bear out her willingness to accept approximations. This rule stipulates that at least some propositions based on induction can be presumed as true *or* approximately true, “until yet other phenomena make such propositions either more exact or liable to exceptions” (Newton 1999, 795). The published version of Du Châtelet’s translation replaces the reference to greater exactitude with the possibility that other phenomena “entirely confirm [*confirment entierement*]” the proposition in question (Du Châtelet 1759/1990, 5). Instead of greater exactitude, what is called for is more inductive confirmation, which may or may not make the original proposition more exact.

Drawing a distinction between more and less contingent approximations helps explain some passages where Du Châtelet seems dissatisfied with theories that employ approximation. For example, George Smith suggests that her *Principia* commentary takes a negative stance on Newton’s simplifying assumption that the moon’s orbit is circular (2022, 295–97). Supposing Smith’s reading of these passages is correct, Du Châtelet’s employment of a “standard of mathematically exact solutions” need not imply a general opposition to approximation (Smith 2022, 296).<sup>7</sup> I take Smith to agree that the problems raised by this particular simplifying assumption need not generalize to all cases of approximation.

Du Châtelet links the methodology of natural philosophy with that of mathematics. For she defends what we might now call quasi-empirical methods in mathematics. To take one example, even elementary arithmetical operations such as division employ hypothetical rather than strictly deductive reasoning (1740, 81).<sup>8</sup> At the time, the division algorithm had

neither been precisely stated (with the tools of first-order logic) nor rigorously founded (in a theory of the integers). So this conclusion may have been prudent. She also considers Leibniz's solution to the brachistochrone problem. This is the problem of finding the curve between two points along which a point mass descends in the shortest time.<sup>9</sup> Du Châtelet calls Leibniz's cycloid solution a "paradox"—we'd expect the shortest path to be a straight line—but accepts the result as "demonstrated" (369). So she may take Leibniz's solution as a well-founded mathematical hypothesis that awaits more rigorous proof (compare 344).

#### **4. Conclusion**

Near the beginning of the *Institutions*, Du Châtelet presents mathematics as a crucial "means" to the foundations of physics (1742, 12). In keeping with this, her discussions of mathematics are primarily focused on its role for inquiry into nature. When she discusses advances in pure mathematics, such as discoveries about the cycloid or Newton's method of first and last ratios, she focuses on their applications. But her account of mathematics and its objects is not derived from empirical science. Instead, this account is embedded in a metaphysical and epistemological system. This background informs Du Châtelet's position that while mathematical objects are not fundamental, they play a privileged role in our knowledge of the world.

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## Notes

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<sup>1</sup> More than half of her posthumously published *Principia* commentary is a mathematical *Solution Analytique* providing calculus solutions to problems from Newton's work (Smith 2022). She also wrote but did not publish a treatise on optics (traditionally grouped under applied mathematics), one manuscript of which is published as Du Châtelet (2017). Further unpublished manuscripts, currently in private hands, apparently deal with mathematical topics such as the principles of arithmetic, conic sections, and Book I of Euclid's *Elements*.

<sup>2</sup> She rejects material atoms, whether these are understood as metaphysically or physically indivisible entities (Du Châtelet 1740, 131; 1742, 107; 199; 200). Yet she grounds bodies in fundamental non-composite and "non-extended" substances that are true unities, analogous to the number one (1740, 132–33). So some finite created beings, though not bodies, are substances in an unqualified sense (71; Gireau-Geneaux 2001). Our epistemic access to fundamental substances is limited, however, and they don't figure in her account of mathematical cognition.

<sup>3</sup> Antoine Arnauld's influential *Nouveaux éléments* exemplifies the pitfalls of this Cartesian approach. For example, he states that Euclid's parallel postulate has "enough clarity" to be assumed as an unproven axiom (1683/2009, 361). Indeed, 'straight line' need not be defined because this "idea is very clear in itself and...all men conceive the same thing by this term" (357–58).

<sup>4</sup> This phrase is deleted from the revised 1742 edition. Du Châtelet also deletes all references to the *essence* of a particular trapezoid, and repeatedly replaces *following from* ("*les attributs découlent*") with *dependence* ("*les attributs dépendent*") (1740, 72; 1742, 76). Why these changes? Her official view is that the attributes or *propria* of a substance follow logically from its essence alone. But a token trapezoid is not a substance, and is partly mind-dependent. Perhaps additional assumptions—for example about space as grounded in relations among substances—are needed for geometrical proofs about the trapezoid. She now speaks of the given properties of a figure as necessary conditions for solving a geometrical problem ("without [them] it would be impossible to solve the problem," 1742, 67), but leaves open whether these conditions are sufficient.

<sup>5</sup> For a contemporary view on which consistency is enough, compare Balaguer (1998, 48–75). Du Châtelet's characterization of actual mathematical objects as *real* may indicate that actuality, on her understanding, can connote concreteness—roughly like the German '*Wirklichkeit*.' As might be expected from the Latin etymology, in early modern French to be '*actuel*' could mean acting or exercising a power.

<sup>6</sup> In the terminology of recent philosophy of science, these examples involve not only abstraction (leaving out some relevant features, such as air resistance), but also idealization (representing some features, such as mass distribution, in a simplified or distorted way).

<sup>7</sup> I cannot fully consider Smith's reading, which includes an account of Newton's work on the three-body problem and assessments of it by Clairaut and others. Still, the textual evidence Smith cites isn't decisive. Du Châtelet's initial discussion is descriptive, not explicitly critical (1759/1990, 98). She accepts cases of approximation where readers can "see" Newton's assumptions, and focuses her criticisms on his failures to make explicit underlying reasoning and evidence (104). So she need not generally reject his use of approximation.

<sup>8</sup> On 'quasi-empirical methods' in mathematics, see Putnam (1975, 60–78). Historical examples Putnam discusses include infinitesimals as postulates or fictions (in Leibniz's calculus), and the postulation of real numbers (in the analytic geometry of Fermat and Descartes). It's also relevant for reading Du Châtelet that, according to Christian Wolff, arithmetic and geometry use hypotheses (1726, § 127; 1724, § 112). Wolff adds that this helps justify the use of hypotheses in philosophy. Later, Kästner and Kant also seem committed to mathematical hypotheses (Kästner 1758, 17; Kant 1980, 29:51–54).

<sup>9</sup> See Goldstine (1980, 30–47). Descartes famously called non-algebraic curves such as the cycloid 'mechanical,' took them to be generable only by separate motions with no exact relationship, and officially excluded them from geometry (Descartes 1964–76, II:307–45; Vuillemin 1960; Grosholz 1991; Bos 2001, 335–96). Du Châtelet

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does not share this negative attitude towards non-algebraic curves, which she considers important for both mathematics and physics. The cycloid, for example, deserves a “whole treatise” of its own (1740, 364). Her discussion cites Wallis (1659), which includes Wren’s solution of the problem of arc length for the cycloid, and her divergence from Cartesianism may be influenced by Wallis’s more pragmatic attitude.