Logical Relations between Pictures

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June 25, 2006

Journal of Philosophy, 2005

Abstract

An implication relation between pictures is defined, it is then shown how conjunctions, disjunctions, negations, and hypotheticals of pictures can be formed on the basis of this. It is argued that these logical operations on pictures correspond to natural cognitive operations employed when thinking about pictures.

Introduction

It is generally assumed that logical relations are relations between sentences, logical formulae or propositions. All of these are linguistic objects; obviously in the case of sentences or formulae, plausibly so in the case of propositions if these are conceived of as collections of sentences 'saying the same thing'. The linguistic or descriptive mode is one major way of representing the world, but not the only (and perhaps not even the most important) one. Another major mode representation is depiction. It represents not by sentences or formulae but by paintings, drawings, sketches, engravings, maps, diagrams, or photographs.

To the best of my knowledge no one has ever inquired into the logical relations between depictions. This is peculiar, for assuming that logic is a general theory of certain fundamental relations between our representations of the world it is hard to see why it should only apply to one class of representations and not to another one. I think there are two main reasons for this peculiarity.

First of all it is often assumed that logical relations could only hold between objects with a transparent syntactic structure. If implication is is seen to be a relation between two formulae, holding if it is possible to transform the first into the second using a fixed set of rules (as in a natural deduction system), then the restriction to descriptive forms of representation is obviously implied. Paintings, drawings and so on do not have a fixed syntactic structures; they are not composed from a fixed set of primitive symbols by a collection of formation rules.[1]

Secondly, common opinion holds that logical relations are only applicable to truth-bearing items. If implication semantically understood to be the relation holding between two items such that whenever the first is true, the second must be true as well we can only consider extending this to depictions if we assume that these, like descriptions, can be true or false. There are a number of problems associated with this idea, as noted by Fodor.[2] When is a picture of a cat on a mat true? If there is some cat on some mat? Or does it have to be a fat cat, as in the picture? Or the very cat pictured? Does it have to be the very same mat? The difficulty of answering these questions does not let pictures appear as the most natural truth-bearers. But in this case there cannot be an implication relation between them, semantically understood.

Both of these assumptions — that implication relations only hold between syntactic items, and only between truth-bearers — are false. A perspicuous demonstration of this has been given in Arnold Koslow's development of a structuralist theory of logic.[3] He defines an implication structure as *any* collection of objects (which have to be neither syntactic nor truth-bearing) on which an implication relation obeying a set of Gentzen-style conditions can be established. Logically complex objects are then defined in terms of this relation. The conjunction of two objects A and B, for example, is taken to be the weakest object C such that it implies A and it implies B. (By 'weakest' we mean that if any other object also implies A and B, it will also imply C.)[4] It then remains to be demonstrated that the objects so defined really have the properties ascribed to logically complex objects such a conjunctions, disjunctions, negations and so on.

One peculiarity of Koslow's system is that collections of objects are not usually logically closed. Frequently logical complexes of objects from the collection will fail to be included in it. For example a collection with only A and B in it will fail to contain a conjuction of the two if they are not mutually entailing. This is due to the fact that the structuralist theory

[1] Indeed Goodman (1976, 225-232) takes this to be the fundamental distinction between linguistic and non-linguistic systems of representation.

[2] (1975, 174–194).

[3] (1992; 1999).

[4] This account cuts a number of corners. For the full details the reader is referred to Koslow (1992).

defines the implication structure on a fixed set of objects; logical complexes are then identified with particular objects from this collection. This is no fundamental limitation, however.[5] The collections can always expanded so as to include the required logical complexes.

In the following I want to use the structuralist theory of logic to develop an account of logical relations between pictures. I will describe an implication relation between pictures and argue that it obeys the conditions mentioned by Koslow. It is then possible to give precise definitions of conjuctions, disjunctions, negations etc. of pictures. It will also turn out that these logical operations are closely related to or even identical with basic cognitive operations we naturally employ when thinking about pictures. Before this can be done, however, it is necessary to dicuss two preliminary matters: the nature of the type-token distinction in the case of pictures and the relationship between a picture and its parts.

1 Preliminaries

1.1 Pictures and paintings

Pictures differ from paintings as propositions differ from sentences. Paintings (as well as drawings, sketches, engravings etc.) and sentences are tokens: spatio-temporally located physical objects. Different paintings can show the same picture, and different sentences can express the same proposition. Pictures and propositions are what particular sets of paintings or sentences have in common, they are therefore not tokens but types.[6] Relative to some set of conventionally defined criteria we regard certain paintings as showing the same picture, and certain sentences as expressing the same proposition. What these criteria are in each particular case is notoriously hard to define, and it is very likely that no analysis of the type-token relation in terms of necessary and sufficient conditions we give will ever be satisfactory. This is, however, not a problem the present discussion will address; we will simply take the type-token distinction for visual representations as primitive.

In discussing the logical relations between visual representations it seems evident that we should concentrate on types (on pictures) rather than on tokens (on paintings). Although it would be foolish to expect an implication

[5] In fact it might be important to allow for the possibility that logical operations are not always defined for all items under consideration. See Scott (1973, 1974).

[6] Whether to conceive of pictures and propositions as abstract objects or in terms of naturalistically more acceptable constructions from paintings and sentences is of little consequence for the present discussion.

relation between visual representations to be in every respect similar to the implication relation between propositions familiar to us from logic, this is certainly a plausible point of departure for investigations into implication relations with other kinds of relata. As such we want to conceive of an implication relation between visual representations as a relation between what various collections of such representations 'showing the same thing' have in common, rather than as a relation between the physical objects which are the basis of the showing or representing. We will therefore look at logical relations between pictures, not between paintings.

Note that due to the greater abstractness of pictures there are some questions concerning them which do not have definite answers, even though the same questions asked about a painting instantiating them do have such answers. If we consider Botticelli's 1489 painting of the *Annunciation* (the tempera-on-wood object in the Uffizi) we can ask e.g. what kind of red pigment was used for Mary's cloak, or what the diameter of her halo is. The same cannot be asked about the *picture* of Botticelli's 1489 *Annunciation*, as the examination of various instances of the picture (a copy painted in oil on canvas, a postcard, an image on a computer screen) delivers radically different answers. This phenomenon can also be observed if the speak just of the picture of the Annunciation, rather than of Botticelli's. Whereas it makes perfectly good sense to ask about either the painting or the picture of Botticelli's *Annunciation* whether the angel approaches Mary from the left or the right, the same question is meaningless when asked about the picture of the Annunciation as such. (In the majority of the cases the angel comes from the right, but not always). It can therefore be the case that we have a precise idea of what a picture depicts (Mary being visited by an angel) without having a fixed opinion on the spatial arrangement of some of its constituents.[7]

It is interesting to note in this context that Daniel Dennett assumes that 'the rules of images in general' (where photographs and paintings are subsumed under images) exclude the lack of specificity just indicated.[8] It may be the case that the rules of *paintings* in general forbid such underdetermination regarding colour, size, composition of parts etc. but appears to be inadvisable to extend this to images or pictures.[9] Of course one would want to assume this if, as in the case of Dennett, the plan is to use this 'rule' to ar-

[7]The same considerations apply when comparing a particular inscription of the fifth commandment, the proposition expressed by it and an arbitrary proposition forbidding murder.

[8]Dennett (1981, 55).

[9]For an argument that in fact such underdetermination can be present in pictorial *tokens* see Tye (1993, 357–360).

gue that mental representations are description-like, rather than image-like. Dennett argues that since an imagined tiger has an undetermined number of stripes the representation of the tiger must be like a description (which is also not forced to specify the number of stripes) and not like an image (which has to depict the tiger with some definite number).[10] I think, however, that if we consider pictures instead of paintings this allegedly crucial difference in determinacy between description- and image-like representation disappears. If there is *any* difference between descriptions and images Botticelli's *Annunciation* belongs to the latter. Yet as we have just seen there are various questions about this image which do not have definite answers. That Botticelli had to give the Virgin's halo some size in the painting he painted does not imply that in the picture he thereby produced her halo also has a definite size. Descriptions and depictions, it turns out, are both forms of representation which admit of underdetermination.

1.2 Parts of pictures: subpictures, constituents, and parts

What are the parts of a picture? The question is not as innocuous as it sounds. The mereology of ordinary objects is well-developed, but pictures are no ordinary objects. First of all they are not spatio-temporal, as we just saw. Secondly they are structured: they are not like a heap of grain or a puddle of water the identity of which is preserved under various rearrangements of their parts. Pictures have parts which are put together in a certain way: if we destroy the order of the parts the picture is gone. Thirdly pictures are representational. A lego toy, a molecule, a pattern of tiles all have structure, but they do not usually represent.

Whilst mereology usually deals with objects in time and space, there is no *prima facie* impossibility in extending the 'part of' relation to other kinds of things.[11] Recent mereological investigations have also started to tackle the second peculiarity,[12] while the third still remains to be addressed.

Some pictures have parts which are pictures themselves. A picture of Napoleon with a hat has a part which is the picture of a hat. So much is uncontroversial. But what about parts of pictures which are not themselves pictures? Here we are faced with two alternatives. Firstly we can conceive of pictures as arrangements of atomic coloured pixels of arbitrarily small size

[10] Dennett (1981, 55).

[11] For example one might regard the prime factors of a number as its parts. An application of mereology to such categories as properties or states of affairs can be found in Meixner (1997).

[12] Simons (1987, 324).

on a plane.[13] Then any pixel and any fusion of an arbitrary collection of pixels from the picture of Napoleon's hat will be a part of the picture, in the same way as each H_2O molecule and any fusion of an arbitrary collection of such molecules will be a part of a given puddle of water. Secondly, we may think of the hue, saturation and brightness of the colour of Napoleon's hat, its outline, the shading and so on as parts of the picture of the hat. It think the second conception is to be preferred to the first. Here is why.

Consider the sense in which states of affairs can be taken to have parts. It is straightforward to argue that the state of affairs that John loves Becca has John as a part. But it is equally straightforward to argue (by the transitivity of parthood) that John's brain is part of the state of affairs that John loves Becca. Metaphysicians generally agree that there is an important difference between these two kinds of parts, a difference which is sometimes expressed by calling the first but not the second kind 'constituents' of states of affairs. It is important to see what the difference between these two kinds is. A state of a affairs is a part of the world we pick out via a certain linguistic description ('John loves Becca', 'John liebt Becca', Ljb etc.) and to which we ascribe a certain structure mirroring the form of the part of language we use to pick it out (the standard analysis is that the state of affairs that John loves Becca consists of two individuals and a dyadic relation). The constituents of the state of affairs are now precisely those parts which we use in conceptualizing this bit of the world; the elements which we take to correspond to the bit of language we use to identify it. But the mere parts (John's brain as opposed to John) are just *any* parts of that particular bit of the world we happen to be talking about, whether they take part in our conceptualization or not. In analyzing a state of affairs we therefore have to concentrate on its constituents, on the parts we actually use in thinking about this part of the world, rather than on any arbitrary collection of parts it happens to have. States of affairs are parts of the world which the human mind has sliced, and in investigating them we have to concentrate on the cuts the mind has made, rather than on any other fissures we may otherwise observe in the material.

A very similar situation confronts us in the case of pictures. These too

[13] Note that this conception does not just confuse pictures with paintings. The picture would be taken to consist of pixels in the same way in which a water molecule consists of hydrogen and oxygen atoms. Different water molecules will have quantitatively distinct atoms as parts, but they will all belong to the same type of atom. In the same way the pixels, the parts of a picture can be different physical objects in the different physical realizations of a picture (they can be fragments of paint, drops or ink or pieces of coloured glass) but still all belong to the same type of pixel. A pixel, like a particular kind of molecule, is no token but a type.

are human creations. As such the outline, saturation, brightness, hue etc. of a picture are more important in investigating them than arbitrary collections of pixels which happen to be parts of the picture. A picture is a representation which has been constructed by combining a certain outline with certain colours, certain shadings and so on rather than a collection of pixels which has been put together bit by bit. A collection of pixels from a picture of Napoleon's hat tells us no more about its rôle as a representation than the part of the state of affairs that is John's brain tells us about its rôle as a part of the world picked out by a particular linguistic description.

In the following investigation we will therefore concentrate on constituents of pictures, rather than on their parts. Neither constituents nor parts are pictures themselves; if we want to refer to pictures which are contained in pictures we use the term 'subpicture'. Note that constituents of pictures — unlike parts or subpictures — are not objects which can exist on their own: we cannot present an outline without a colour, or a colour without a shape. The constituents of a picture are ontologically dependent on one another.[14] Furthermore, as the reader is invited to check all three kinds of parts of pictures are subject to the standard mereological axioms; the subpicture-, the constituent- and the part-relation are reflexive, transitive and antisymmetric.

One complication we should note, however, is that collections of pixels from a picture can fail to be either a part or a subpicture. They fail to be a part because they are a picture themselves, and they fail to be a subpicture because they are not contained in the picture.

To see how this can be the case consider the closely related case of sentences. Not every part of a sentence (string of symbols contained within it) which is a meaningful expression is also a constituent of that sentence. It must also be a meaningful expression *which is constitutive of the meaning of the entire sentence*. For example in the sentence 'to you fall the rewards' the part 'you fall' is meaningful, but its meaning is not part of the meaning of the whole sentence and thus not a constituent.[15] Similarly, to use an example of Pinker's, the expression 'sex with Dick Cavett' is a constituent of the sentence 'the TV show discussed sex with Dick Cavett' only on one of its two possible readings.[16]

A similar situation can arise with pictures by using clever cropping. In

[14] The same is true of constituents of states of affairs: individuals depend on properties they instantiate, properties depend on individuals as instantiators. They are abstractions from states of affairs, but not things which could exist independently, outside the context of a state of affairs.

[15] This example is from Sober (1976, 122).

[16] Pinker (1994, 102–103).

this case a part of the picture is cut out which is indeed a picture itself, but none which could be discerned in the original picture with its original surroundings (and is therefore not a constituent). It is no more constitutive of the first picture than 'sex with Dick Cavett' is constitutive of the above sentence where 'sex' is the entire noun phrase and 'with Dick Cavett' a prepositional phrase (as opposed to the typographically identical one where both 'sex' and 'with Dick Cavett' are constituents of the noun phrase).

2 Implication relations between pictures

There is a natural way of conceiving of an implication relation between pictures in terms of the relation of pictures and their subpictures. We might argue that in the case of two pictures, one of which shows a still life with fruits, and the other only a peach from this still life, the former implies the latter. More generally we could say that a picture implies all its subpictures.[17] Elliott Sober in his treatment of logical operations on picture agrees and claims that 'the pictorial analogue of implications is containment'.[18] He also argues that containment is the *only* pictorial counterpart of implication. If this was indeed the case there would not be much of interest in the study of implication relations between pictures. There would only ever appear one name of a picture on the left of any pictorial implication sign as only single pictures could imply anything individually, but never a group of them collectively. Moreover, implication would typically be unidirectional, as the item on the left of the implication-sign would contain more information than the one on the right.[19]

Neither of these two characteristics (which are independent of another) is true of the familiar implication relation between propositions. Virtually all interesting implications have more than one premiss, and while many are of the 'information-losing' variety, in many other cases *implicans* and *implicatum* are equivalent.

[17]Note that in this is an important respect in which pictures differ from sentences: 'The book is red or the book is heavy' implies neither of its two proper subsentences.

[18]Sober (1976, 122).

[19]Apart from the case of a picture being its own subpicture, which is always guaranteed by reflexivity. Note that there is also the case of a picture containing a near identical subpicture, like the label of a cheese showing a monk eating cheese on the label of which there is a monk eating cheese and so on. Assuming finite complexity of pictures this regress has to stop somewhere (the label on the last cheese is just an array of pixels) and so this is *not* a case of a picture containing itself as a subpicture. If there are, say, five iterations then the largest picture will contain a weak part with only four iterations, which is obviously not identical to itself.

Examples of each type can be conveniently summarized in the following table:

	ONE PREMISS		MULTIPLE PREMISSES	
UNIDIRECTIONAL	$A \vdash A \vee B$	[1]	$A \to B, A \vdash B$	[3]
BIDIRECTIONAL	$A \to B \vdash \neg B \to \neg A$	[4]	$A, B \vdash A \wedge B$	[2]

If the pictorial analogue of implication was indeed containment, all implications between pictures would be of the one-premiss unidirectional variety (type [1]). This, however, is not the case. All four possibilities given in the above table have pictorial analogues.

Let us first of all look at pictorial implication with multiple premisses. For an example of the bidirectional case (type [2]) consider the following. Assume there is a fresco at the end of a room which we are only allowed to view from a certain distance. Unfortunately from our point of view there are always two columns blocking our view of parts of the fresco. We can move around the room to see any part of the fresco, but we can never observe the whole fresco without the columns. Now it is clear that we can piece together the appearance of the complete fresco in our mind from the parts we were able to observe. We could do a similar thing with photos we have taken from different points of view: cut out the columns and then arrange the fragments in such a way that we get a continuous picture.

I want to argue that the parts of the fresco thus observed collectively imply it. Not only will a picture imply any of its subpictures, a collection of pictures will also imply one which is the result of putting all of them together in a certain way. Drawing implications is not just reducing the information present in the premisses (‘The book is red and square’ therefore ‘The book is red’), but equally combining into a compound (‘The book is red’, ‘The book is square’ therefore ‘The book is red and square’).

Now take a case with multiple premisses which is unidirectional (type [3]). In this case two pictures imply a third one which contains less information than the two of them together. Assume we have two pictures of the same statue taken from two different positions, A and B. In some cases it is possible to work out what a view of the statue from some position C between A and B would look like, solely on the basis of the two pictures from positions A and B. Then we would also want to say that the picture from C is implied by the pictures from A and B. It would then contain less informational content than the pictures which imply it.

Considering this example shows that it is unhelpful to conceive of pictorial implication in terms of 'cut and paste' operations. These can account for unidirectional implications with one premiss (where a subpicture is 'cut out') and bidirectional implications with multiple premisses (where the conclusion is 'pasted together' from the premisses). But 'triangulating' a picture from two other ones in the way just sketched is not covered by such operations, nor are cases of bidirectional pictorial implications with one premiss (type [4]). The most straightforward (if slightly boring) example is just the relation every picture bears to itself (the pictorial equivalent of $A \vdash A$). The fact that there are no more interesting examples of bidirectional pictorial implications with one premiss (something like pictorial equivalents of $A \rightarrow B \vdash \neg B \rightarrow \neg A$) is due to the analogy between pictures and propositions described above. Pictures are collections of visual tokens which show the same, propositions are collections of linguistic tokens which say the same. Logical formulae 'say the same' iff they are logically equivalent, so $A \rightarrow B$ and $\neg B \rightarrow \neg A$ express the same proposition. Had we constructed the above table of different kinds of implication with propositions instead of logical formulae it would have been immediately obvious that the only example of a bidirectional implication relations with one premiss would indeed have been of the form 'A implies A'.

I have argued that each of the four kinds of implication given in the above table have a pictorial analogue. I also want to claim that four examples I have described (subpicture implication, the fresco case, the statue case and the case of pictures implying themselves) all involve *a single pictorial implication relation*. What is my argument for this? The most elegant argument would be to reduce the four examples to one, by defining three in case of a fourth. Subpicture implication (type [1]) seems to be an attractive candidate for this, as it can be used to define type [4] (if everything is a subpicture of itself) as well as type [2] in the following way:

$P_1 \ldots P_n$ [2]-imply Q iff for every R, if P_1 [1]-implies R (i.e. if R is a subpicture of P_1), and $\ldots P_n$ [1]-implies R, then Q [1]-implies R.

Unfortunately, it is not possible to define type [3] in terms of type [1] as well, nor is there any other of the four examples which could be used to define all the other ones. We therefore have to use another strategy to argue that the four examples are all special cases of a single pictorial implication relation. Consider the following definition of a pictorial implication relation $\rightharpoonup$:

$P_1 \ldots P_n \rightharpoonup Q$ iff the information contained in Q is properly or improperly included in the information in $P_1 \ldots P_n$.[20]

[20] John Corcoran has developed a theory of implications for *propositions* based on the

First of all note that there are two distinct senses of understanding the informational content of a picture. We might conceive of it as objective information which is exclusively a feature of the picture, or as subjective information, which depends on the sensory and cognitive apparatus of the perceivers. Two different pictorial tokens might contain the same objective information but different subjective information. An example of this is the case of a photograph and its negative: we might be able to extract more information from one rather than the other, though the objective informational content of the two is exactly the same. For the purposes of this paper I will restrict myself to objective informational content. The information a picture contains is therefore taken to be identical for all different possible perceivers considered.

Now the informational content of a subpicture thus perceived is obviously properly included in the bigger picture of which it is a part. All the parts of a fresco contain the same information as the fresco they jointly constitute.[21] If we use two pictures to generate an intermediate perspectival view we lose some information in each of them, so the information present in the intermediate view is properly included in the information contained in the pictures which gave rise to it. Each picture contains the same information as itself. Thus it seems as if this definition in terms of information-content subsumes the four examples of pictorial implication given.

I am aware that my appeal to the information-content of pictures involves a considerable amount of hand-waving. There is at present no satisfactory worked-out theory of the information-content of visual representations (nor, one should hasten to add, is there workable a theory of semantic content on an information-theoretic basis). The present paper is certainly not the place for coming up with such a new theory. All I am doing here is noting the intuition that if we take a colour picture and cut off a bit, or transform it into a black-and-white picture, the resulting picture contains less information than the original, while the results of, say, mirroring it or inversing the colours would

very same idea. See Corcoran (1998).

[21] A possible worry one might have with this is the following. Assume two parts of a fresco, A and B, each show a part of a flower. The information about how many petals the flower has is neither present in A nor in B. Indeed looking just at A and B we do not even know whether they are parts of the picture of the *same* flower. But this information is present in the entire fresco, consisting of A and B put together. So the entire fresco, which is jointly implied by A and B contains more information A and B separately, contrary to the definition, which claims that the information-content in the *implicatum* is always smaller than or equal to that of the *implicans*. The worry can be answered by referring to the distinction between conjunctions and concatenations introduced below. Only the concatenation of A and B contains the information about how many petals the flower has, but not the conjunction. As only the conjunction, not the concatenation is implied, the problem disappears.

not affect the amount of information contained. A theory of the information-content of pictures should be able to account for this intuition, and once such a theory has been developed we can give a clearer account of the background of the definition of pictorial implication in terms of information-content given above. The precise details of the theory do not matter much for our present purposes, as long as there is *any* reasonably systematic way of accounting for the intutions just given (which seems to me beyond reasonable doubt).

The implication relation between pictures just presented is construed as a mono-categorial implication relation: the signs on either side of the implication sign denote items of a single type or category (namely pictures), in the same way as the objects denoted by the variables flanking $\vdash$ or $\Vdash$ are uniformly formulae or propositions, respectively. However, this is not the only and not even necessarily the most natural way of conceiving of implications between complexes such as pictures or states of affairs. It seems very straightforward to view the implication relation for complexes as poly-categorial. This is because a complex can imply something which is not a complex itself, or can in turn be implied by non-complexes. A picture's constituents are not pictures in turn, but can become so by adequate supplementation; similarly the state of affairs that John is a bachelor might not just imply the state of affairs that he is male, but also its various constituents (the individual John, the monadic property of being male), which are no states of affairs. Conversely, two non-pictures, two constituents, can be put together to form a picture, thus implying it; two constituents of states of affairs (e.g. an individual and a property) can imply the state of affairs resulting from putting the two together.[22]

Although I think that a satisfactory theory of implications dealing with complexes such as pictures or states of affairs should be poly-categorial in the sense just described I will not pursue this idea further in the present paper. To keep matters simple I will assume in the following that the symbols on either side of the pictorial implication-sign denote complete pictures, not their constituents.

It is important to note that the implication relation between pictures from a collection G just defined satisfies all of Koslow's six conditions for being an implication relation:

[22] Of course parts of complexes can also imply other parts: constituents of pictures (e.g. the complex of an outline together with a shading) can have constituents themselves (the outline, and the shading); the property 'loving Becca' (which is a constituent of a state of affairs) implies a further constituent, the individual Becca.

1. REFLEXIVITY — for all A in G, $A \rightharpoonup A$

2. PROJECTION — $A_1, \ldots A_n \rightharpoonup A_k$, for any $k = 1, \ldots, n$

3. SIMPLIFICATION — for all A_i, B in G, if $A_1, A_1, A_2, \ldots A_n \rightharpoonup B$, then $A_1, A_2, \ldots A_n \rightharpoonup B$

4. PERMUTATION — if $A_1, A_2, \ldots A_n \rightharpoonup B$, then $A_{f(1)}, A_{f(2)}, \ldots A_{f(n)} \rightharpoonup B$ for any permutation f of $1, 2, \ldots, n$

5. DILUTION — if $A_1, \ldots A_n \rightharpoonup B$, then $A_1, \ldots A_n, C \rightharpoonup B$ for all A, B, C in G

6. CUT — if $A_1, \ldots A_n \rightharpoonup B$, and $B, B_1, \ldots B_m \rightharpoonup C$, then $A_1, \ldots A_n, B_1, \ldots, B_m \rightharpoonup C$ for all A_i, B_j, B, and C.

Every picture implies itself (this follows from the reflexivity of the subpicture relation), and the same holds if we consider any collection of pictures (thus satisfying projection). Repetition of pictures in a collection does not affect their implications, nor does their order. Pictorial implication is monotonic and sequences of implications can be simplified using cut.

At this point it is instructive to consider another relation between pictures which we might intuitively consider an implication relation of sorts, but which fails to be one, as it only satisfies some of the above conditions.

The individual pictures making up a comic-strip which are usually arranged in a narrative sequence might be taken to imply one another. If we consider a sequence of three pictures, the first showing a hammer striking a vase, the second the vase shattering, and the third the cat sleeping next to the vase waking up it is not implausible to argue that in some way the first picture entails the second, and the second entails the third. The notion of implication in play here is, however, very different from the one studied in logic. While it definitively satisfies reflexivity and projection, and possibly also simplification,[23] permutation fails to be satisfied since the implication in this case also entails a temporal ordering: if we change the sequence of the pictures in the comic-strip the implications will not necessarily be preserved.

[23] This depends on whether we think that a series of repeated pictures in a comic-strip can unproblematically be reduced to a single occurrence.

Dilution also fails: if one picture implies the following it is not always the case that this implication is preserved when adding another one. The picture of the vase shattering implies that of the cat waking up, but if we interpose a picture between the two which shows that the shattering of the vase was only an image on a TV screen in the same room as the cat then the final picture will no longer be implied. Finally cut is not satisfied. If we 'cut out' a sufficiently long sequence between pictures in a comic-strip it is not always the case that last picture before the cut will imply its new successor, which is the first picture after the cut.

Unlike this 'comic-strip implication' the implication relation between pictures based on informational content described above behaves structurally exactly like the implication relation familiar from logic. We will now consider how the different logical operators (conjunction, negation, disjunction and the hypothetical) between pictures can be defined on the basis of this implication relation.

2.1 Conjunction

For any collection of pictures G we define the conjunction $Conj(A, B)$ of two pictures A and B from G to be the weakest picture in G such that $Conj(A, B) \rightharpoonup A$ and $Conj(A, B) \rightharpoonup B$. By saying that it is the weakest picture we mean that if for any other picture X from G, $X \rightharpoonup A$ and $X \rightharpoonup B$ then $X \rightharpoonup Conj(A, B)$.

It is instructive to consider the relation of the *logical* operation of pictorial conjunction with the *spatial* operation of concatenation. Elliott Sober regards concatenation as a conjunction operation,[24] at least when considering typical cases of concatenation.[25] Sober's notion of conjunction is, however, fundamentally different from the one developed here. He employs no framework for defining conjunction directly at the level of pictures, but only via their linguistic descriptions. Sober considers a representation function I such that for every picture p, $I(p)$ is a sentence specifying the information p provides.[26] It is then argued that an operation $+$ on pictures is the equivalent to the conjunction operation on sentences in case $I(p + q) = I(p) \wedge I(q)$. Neglecting cases of clever blending the operation $+$ can then be identified

[24] Sober (1976, 122)

[25] Concatenation 'fails to exactly exemplify pictorial conjunction' (121) because of the reverse of clever cropping (which we might want to call 'clever blending'). If we concatenated two pictures and these would blend into a new picture distinct from the original ones the result would not be a conjuction, as the conjuncts are no constituents of it. See also Howell (1976, 160–161).

[26] Sober (1976, 112).

with concatenation.

Apart from the methodologically unattractive feature of having to switch systems of representations first in order to make sense of logical operations on pictures, Sober's account also faces internal difficulties, as noted by Howell.[27] If we have two pictures each of which shows two apples next to one another then part of the information each picture provides is that there is exactly one apple to the right of the leftmost apple. Concatenating the pictures so that they show four apples in a row this is no longer the information given, for now there are three apples to the right of the leftmost one.

Of course Sober could defend himself against this charge by modifying his claim and asserting that pictorial conjunction is not the equivalent of $\wedge$ but of a more complicated truth-functional operation called conjunction' which only implies some, but not all of its conjuncts'.[28] Whether this defence is successful depends on the properties conjuction' turns out to have in the end. More problematic seems to be that if we go down that route we are essentially answering a different question: instead of developing a theory of logical operations on pictures we now inquire which of the claims of its subpictures a picture preserves. This is due to Sober's reluctance to deal with pictures directly and his concentrating instead on their linguistic equivalents.

The main reason, however, why we cannot identify conjunction with concatenation is that conjunction is unique, while concatenation is not. If we have two pictures there are various ways in which we can concatenate them in the plane: putting one on top of another or the other way round, one to the left of the other or the other way round, and so on. The difference is not that conjunction is a 'purely mental' operation on pictures while concatenation would actually have to move physical pieces around. Both operations deal with pictures, not with paintings, and therefore do not deal with physical objects. To see that the same difference arises in the case of two purely mental operations compare the *mere combination* of some ideas (Paul, Peter, is taller than) and their *combination in a judgement* (Peter is taller than Paul, Paul is taller than Peter): the first operation just specifies that the items in question are put together, the second determines how exactly this is to be done.

Conjunction and concatenation are therefore two distinct cognitive operations on pictures. They can of course be applied in succession: we can form the conjunction of some concatenations, and we can also concatenate conjuncts. This latter is of particular interest when considering logically complex

[27] Howell (1976, 161)

[28] A straightforward way of accounting for the phenomenon of clever blending would be to say that conjunction' is non-monotonic.

operations on pictures below. Concatenation on its own does not entail any minimality conditions (the sequence of words 'Peter plays well' qualifies as a concatenation of the words 'Peter' and 'plays'), but concatenation *of conjuncts* inherits the minimality condition from the definition of conjunction: the resulting object must be the weakest which can be concatenated from the elements of the conjunction.

While concatenations of pictures are obviously pictures, conjunctions are as well, although (unlike concatenations) their constituents do not stand in a determinate spatial relationship. It makes no sense to ask of the conjuction of the pictures of a cat and a mat whether the first is above the second or the other way round. But this does not keep it from being a picture (although a relatively abstract one) in the same way in which 'a picture of the Annunciation' is a picture, although there is not definite answer to question regarding which side the angel is coming from.

Conjoining pictures is a cognitive operation which is particularly important when creating paintings. If we are commissioned to paint an Annunciation for a chapel or a cat-mat picture for a philosophy textbook the basis for the design is the weakest picture implying all the necessary subpictures (Mary, the angel, a lily; a cat and a mat), i.e. the conjunction of these subpictures. Whilst designing the picture we then decide how to put these conjunct into a spatial relation with one another, that is, we form a concatenation. On the basis of this we can then proceed to execute the painting.

2.2 Disjunction

For any collection of pictures G the disjunction $Disj(A, B)$ of two pictures A and B from G is the weakest picture in G such that for any T in G, if $A \rightharpoonup T$ and $B \rightharpoonup T$, then $Disj(A, B) \rightharpoonup T$.

The disjunction of two pictures is therefore the weakest picture which implies what each of the two disjuncts implies. While the logical operation of conjunction on pictures is central to the creation of paintings, disjunction is central to their comparison, for it gives rise to pictorial abstraction.

Assume we have two identical pictures of a peach, with the difference that the first shows the peach on a table, while the second shows it on a chair. The disjunction of the two is then just the subpicture of the two which shows the peach. We have therefore abstracted the 'common element' of the two pictures. Although not every pair of pictures has a disjunction, it is still plausible to regard forming the disjunction as a widespread and natural cognitive operation on pictures. Its precise form depends on the kind of identity conditions we assume for pictures. The strictest conditions would demand a pixel-by-pixel correspondence between pictures, while a more flexible approach might regard two pictures of the same thing as identical, even though they might not have a single pixel in common. On the latter understanding we can see how the notion of the disjunctions of pictures stands behind all attempts to give an account of a particular pictorial motive across a series of pictures. If we e.g. attempt a study of the depiction of windmills in the nineteenth-century English painting the raw material to consider will be the disjunction of a particular set of paintings, namely those subpictures which are pictures of windmills and which all of a particular collection of paintings have in common.

2.3 Negation

For any collection of pictures G the negation $Neg(A)$ of some picture A in G is the weakest picture in G such that for any picture B in G, $Neg(A)$ together with A implies B.

This definition exploits the fact that something together with its negation implies anything whatsoever. The conjunction of a picture and its negation will be a contradictory picture; a picture of which everything is a subpicture. Whether there are contradictory pictures is a contentious issue,[29] but my account can remain neutral on this point. As collections of objects are not necessarily closed under implication relations there is no need to assume that any such 'big picture' will indeed exist.

If our gallery G is rather boring and contains only three pictures of different pieces of of fruit: P, the picture of a peach, Q, that of a quince, and R, that of a raspberry then $Neg(P)$ will just be $Conj(Q, R)$. Once a clear notion of implication between pictures is in place there is therefore nothing inherently mysterious about negative pictures.[30] We do not have to assume that negative pictures depict what is not the case, that they are all crossed by a red diagonal line, or that there is a more subtle hint of negativity present in them. Negative pictures are just pictures which are implicationally related to others in a particular way.

In fact we apply the operation of negation quite naturally in thinking about pictures whenever we consider a particular part in isolation and compare it to the rest of the picture presently at hand. If we discuss a painting which shows some figures in a landscape, and we want to direct somebody's attention to this very landscape we might say: 'Never mind about the men in the foreground — just concentrate on the rest'. What we are asking him

[29]See Sorensen (2002) for an (indecisive) discussion.

[30]As there is nothing inherently mysterious about negative states of affairs once a mereology has been defined on them. See Meixner (1997, 44–45).

to do is to consider the negation of the men in the foreground, i.e. the largest subpicture of the painting of which the men in the foreground are no part.

The definition of negation also allows us to address a worry connected with the fact that implication relations are easy to come by: every non-empty set can be equipped with one. This profusion of implication relations is particularly evident when considering the dual of an implication relation. For any implication relation $\rightarrow$ its dual $\rightarrow^*$ is defined as follows: $A_1 \dots A_n \rightarrow^* B$ iff for every T, if $A_1 \rightarrow T$, ... and $A_n \rightarrow T$, then $B \rightarrow T$. Now whenever $A \rightarrow B$, $B \rightarrow^* A$.[31] So whenever for two pictures S and T, $S \rightharpoonup T$, it is also the case that $T \rightharpoonup^* S$, where $\rightharpoonup^*$ is defined as

$P_1 \dots P_n \rightharpoonup^* Q$ iff the information contained in Q properly or improperly includes the information in $P_1 \dots P_n$.

To make sense of this implication relation consider a particular case of $\rightharpoonup$, namely subpicture (type [1]) implication. This links a picture to all its subpictures. The dual of subpicture implication will do the reverse, and link it to all the pictures of which it is a subpicture. As this 'superpicture implication' has the same structural properties as subpicture implication (as specified on page 12) we are therefore see that there is some way in which a picture implies not just all the pictures which are parts of it, but also all those of which it is a part. There is no necessary connection between two objects standing in an implication relation and the *implicans* having a greater or equal informational content than the *implicatum*. Similar considerations apply to the other particular cases of $\rightharpoonup$.

Now we might rightly question why the present paper has chosen to study $\rightharpoonup$ rather than $\rightharpoonup^*$, given that the latter is also a perfectly good implication relation. The reason is that $\rightharpoonup$ has a much closer connection with the cognitive operations we employ in thinking about pictures than $\rightharpoonup^*$, and is therefore particularly interesting. This becomes evident in the treatment of negation. Had we chosen $\rightharpoonup^*$ our pictorial implication relation the roles of conjunction and discjunction would have been reversed: disjunction would have formed the basis of concatenation, conjunction would have been connected with abstraction.[32] This, it seems, would have been little more than a change in terminology. Consider, however, what happens if we replace $\rightharpoonup$ by $\rightharpoonup^*$ in the above definition of negation. Let us once again think of $\rightharpoonup^*$ as superpicture implication. Now instead of the negation of a picture being something such that the conjunction of this negation and the picture together have any other

[31]Koslow (1992, 62, 9.1).

[32]Koslow (1992, 115–116).

picture as a subpicture, it is now the case that the conjunction of a picture and its negation are a subpicture of any picture whatsoever. It is not easy to make sense of this intuitively, but but assume there was an ubiquituous picture U in a collection of pictures, something which was a subpicture of every picture in the collection. Then the negation of a picture A could be conceived of as some sort of thing which hooks onto A and at the same time transforms the compound into U.

The important point now is that there is no straighforward conceptual equivalent to negation defined in terms of $\rightharpoonup^*$ we use in thinking about pictures. There is one if we define pictorial negation in terms of $\rightharpoonup$: it is what allows us to isolate particular parts of pictures from others. For this reason $\rightharpoonup$ appears to be a more interesting pictorial implication relation than its dual.

2.4 Hypothetical

For any collection of pictures G containing A and B, the hypothetical $Hyp(A, B)$ is the weakest picture in G such that $A, Hyp(A, B) \rightharpoonup B$.

The hypothetical of two pictures is thus that picture which, together with the first one, implies the second. Again it is easy to find a natural example of hypothetical pictures. Consider the case of reconstructing a picture (a fresco, for example) from an incomplete set of fragments. Let A be a fragment of a fresco and B a photographic reproduction of a detail from the fresco before its partial destruction. Let us also assume that the picture B shows partially overlaps the one A shows. Now if we added a reconstruction of the remaining part to A the resulting complete fresco would imply B. This remaining part therefore fulfils the condition of being the hypothetical of A and B: it is what we have to add to A in order to get B out.

To take another example, consider the picture A of a building from a certain perspective, say, from the north. Suppose we wanted to have a picture B of the same building from the north-west. Various pictures would imply B together with A (a picture of the building from the west, from north-north-west, and so). The weakest of these (if there is one) will be the hypothetical $Hyp(A, B)$.

2.5 Complex operations

It is now apparent that the application of logical operations to pictures defined in structuralist terms yields natural and familiar cognitive operations we use when thinking about pictures. Conjunctions serves as a basis for concatenation (putting pictures together to form a bigger one), disjunction results in abstraction (extracting the common parts of distinct pictures), negation yields isolation (extracting a particular part of a single picture), and forming hypothetical gives supplementation (adding the missing parts of a picture).

But these operations are not just employed on their own, they are frequently combined. Restoring a fresco, for example, entails first forming a series of hypotheticals $Hyp(A, B)$, based on a particular extant fragment A and some conception of what the original B looked like. Subsequently one concatenates the conjunction of the fragments with their hypotheticals to form the completed picture.

A more pedestrian but equally interesting case is constituted by 'spot the difference' pictures frequently found in childrens' magazines. These consist of two nearly identical pictures which differ in a specified number of respects; the goal is to identify all of them. The logical operation required here is that of forming the negation of the disjunction of the two pictures. Taking first all the parts which the pictures have in common, and then identifying the strongest parts which do not imply anything the common parts imply will give us exactly all the subpictures in which the two pictures differ.

Similarly situations can arise in more serious contexts. In a recent case it was argued that a landscape painting by the 19th-century Russian realist Ivan Shishkin to be auctioned at Sotheby's for £700,000 was really a doctored version of a work by the relatively unknown Dutch Martinus Koekkoek, (valued at about £5000).[33]

The only difference between the two paintings is that the presumed Koekkoek shows some figures where the alleged Shishkin shows none; to compensate, the latter one shows a signature (that of Shishkin) which is lacking in the former. These differences are of course the negation of the disjunction of the two paintings. Different parts of this picture are hypotheticals which can be conjoined with the disjunction of the two paintings ('what they have in common') to turn it either into the Koekkoek or the Shishkin. If the signature was copied from another painting by Shishkin it constitutes the negation of all the other subpictures in this other painting which was then conjoined with the negation of a small part of the Koekkoek (namely the part where the signature was going to go). It is evident that the two paintings stand in a variety of logical relations which are, however, nothing but a precise articulation of familiar operations like concatenation, abstraction, isolation and supplementation we use when dealing with pictures.

I hope this paper has shown that the notion of implication between pictures, as well as that of logical operations on pictures can be defined in a

[33]As reported in the *Guardian*, 10th July 2004.

precise way. Moreover, these logical operations are not just interesting from a purely formal perspective but correspond to cognitive operations which we apply naturally when thinking about pictures.

I expect these observations to have implications for both the theory of representation and the philosophy of mind. Regarding the former a theory of logical relations between pictures will help to overcome the unnecessary fixation of philosophical work on representation with linguistic representation by showing how logical relations hold between all kinds of representation, whether linguistic or not. Regarding the latter, much of the discussion in the current imagery debate is based on a more or less sharp division between syntactic and image-like forms of representation. Showing the extent to which logical relations hold for both could be the first step towards a unified theory of mental representation which incorporates both the descriptive and the depictive modes as special cases. Both of these topics will, unfortunately, have to be left as subjects of further investigation.

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