

Centering the Principal Principle

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(penultimate version; see *Philosophical Studies* for the final manuscript)

Abstract

I show that centered propositions—also called *de se* propositions, and usually modeled as sets of centered worlds—pose a serious problem for various versions of Lewis’s Principal Principle. The problem, put roughly, is that in scenarios like Elga’s ‘Sleeping Beauty’ case, those principles imply that rational agents ought to have obviously irrational credences. To solve the problem, I propose a centered version of the Principal Principle. My version allows centered propositions to be objectively chancy.

1 Introduction

According to various versions of the Principal Principle—for example, one formulated by Lewis (1980), one suggested by Hall (1994), and one advocated by Ismael (2008)—agents ought to set their credences in propositions equal to the known chances of those propositions. In slogan form: rational credence is constrained by chance. For example, suppose Susie knows that the chance of a coin landing heads is $\frac{1}{2}$, and suppose Susie has no other information about the upcoming coin flip. Then in order to be rational, Susie must have credence $\frac{1}{2}$ in the coin landing heads.

Typically, all three versions of the Principal Principle are taken to constrain rational *uncentered* credences, where an uncentered credence is a credence in an uncentered proposition. But throughout the literature, these principles are often assumed to constrain rational *centered* credences as well (Elga, 2000; Lewis, 2001; Meacham, 2008; Ross, 2010; Weatherson,

2013), where a centered credence is a credence in a centered proposition like “It is Monday” or “I am Dr. Evil”. So some natural questions arise. Do these principles need to be adjusted, when applied to centered credences? Or do these principles constrain centered credence in basically the same way that they constrain uncentered credence?

As I shall show, all three versions of the Principal Principle face a serious problem when applied to centered propositions: each implies that rational agents ought to have obviously irrational credences. The problem arises in a scenario modeled after Elga’s ‘Sleeping Beauty’ case (2000). According to these principles, rationality requires the agent in Elga’s case to have particular credences in particular propositions. But those particular credences, in those particular propositions, force the agent to have irrational credences in other propositions. So the rational constraints imposed by these principles lead, ultimately, to irrationality.

After presenting the problem, I propose a way out. My solution postulates *centered* chances: chances, that is, of centered propositions. Centered chances are liable to strike many readers as odd: how can centered propositions—like “It is Monday” or “I am Dr. Evil”—be objectively chancy? But as I shall show, centered chances are not as obscure as they initially seem to be: many strategies for explicating *uncentered* chances—chances, that is, of uncentered propositions—can be adapted into strategies for explicating centered chances. For instance, just as uncentered chances can be analyzed using the best system account of laws (Lewis, 1994, p. 480), centered chances can be analyzed using the best system account of laws. And as I shall show, centered chances may even feature in our best physical theories. Centered chances can be used to make sense of probabilistic posits in both contemporary cosmology and the Everett interpretation of quantum mechanics.

So this paper contributes to the literature on credence and chance in several different ways. For starters, I show that a paradox can be derived from a few standard assumptions—concerning centered credences, and concerning the Principal Principle—in the literature on rational credence. After that, I propose a new metaphysical posit: centered chance. Then I take some initial steps towards developing a theory of what centered chance is. I use

centered chances to revolve the paradox, I analyze centered chances using the best system account of laws, and I explain how centered chances might figure in cosmological and quantum mechanical theories.

The structure of the paper is as follows. In Section 2, I summarize the version of the Principal Principle on which I will focus, and I show that centered propositions make trouble for it. In Section 3, I propose a replacement principle which invokes centered chances. In Section 4, I analyze centered chances using the best system account, and I use two physical theories—contemporary cosmology, and the Everett interpretation of quantum mechanics—to further explicate the notion of a centered chance.

2 The Problem of Centered Propositions

2.1 The Principal Principle

In this paper, I focus on the following formulation of the Principal Principle, taken from Lewis (1980, p. 266). Let A be a proposition. Let t be a time. Let Ch be a chance function defined, at time t , over an algebra of which A is a member. Let x be a real number in the unit interval. Let E be any proposition that is (i) consistent with the proposition that $Ch(A) = x$, and (ii) admissible at time t .¹ Then a rational agent's initial credence function Cr ought to satisfy the equation below.

$$Cr(A \mid E \ \& \ Ch(A) = x) = x$$

This is the ‘Principal Principle’.

The problem I raise for the Principal Principle is extremely general: versions of it arise for other principles too. For instance, one version of the problem arises for the ‘New

¹See Lewis (1980) for a discussion of admissibility.

Principle’ (Hall, 1994, p. 511; Lewis, 1994, p. 487), and another version arises for the ‘General Recipe’ (Ismael, 2008, p. 298). And the solution I propose in Section 3 works for those other principles as well. See the appendix for discussion of those principles and the problems which they face.

2.2 The Problem

In this subsection, I show that a problem arises when the Principal Principle is applied to the ‘Sleeping Beauty’ case described by Elga (2000); call it the ‘principal problem’. When applied to the centered propositions invoked in Elga’s case, the Principal Principle implies that upon being woken on Monday, rationality requires being completely certain that it is Monday, even though Monday awakenings are subjectively indistinguishable from Tuesday awakenings. So when applied to Elga’s case, the Principal Principle forces agents to have obviously irrational credences.

Here is the case which leads to the principal problem. Suppose that on Sunday, Susie is told the following. She will be put to sleep that evening. Then she will be woken on Monday. After a brief period of time, she will be told that it is Monday. Then she will be put back to sleep. Later that evening, a fair coin will be flipped. If the coin lands heads, she will be woken on Wednesday and the experiment will be over. If the coin lands tails, she will be given a special drug which erases her memory of the Monday awakening, and she will be woken on Tuesday. Then she will be put back to sleep, she will be woken on Wednesday, and the experiment will be over. Call this the ‘Susie experiment’.

In the remainder of this subsection, I present a fully rigorous formulation of the problem. But by way of preparation, here is a quick summary of the basic issue. The chance of the coin landing heads, both before Susie is told that it is Monday and after Susie is told that it is Monday, is $\frac{1}{2}$. So the Principal Principle implies that Susie’s credence in the coin landing heads ought to be $\frac{1}{2}$, both before and after learning that it is Monday. But that,

in conjunction with a couple other plausible constraints on Susie's credences, implies the following: after being woken up on Monday but before being told that it is Monday, Susie's credence in it being Monday must be 1. In other words, Susie must be completely certain that it is Monday even though her Monday awakening is, from her point of view, just like her Tuesday awakening.

Now for the details. For simplicity, suppose there are four possible ways that the world could be, when Susie wakes up on Monday.

H_m : the coin lands heads, and it is Monday.

H_t : the coin lands heads, and it is Tuesday.

T_m : the coin lands tails, and it is Monday.

T_t : the coin lands tails, and it is Tuesday.

Let H be the proposition that the coin lands heads. So H is $H_m \vee H_t$. Let T be the proposition that the coin lands tails. So T is $T_m \vee T_t$. Let Mo be the proposition that it is Monday. So Mo is $H_m \vee T_m$. And let Tu be the proposition that it is Tuesday. So Tu is $H_t \vee T_t$.

Let $t = 1$ be a time after Susie is woken on Monday, but before she is told that it is Monday. Let $t = 2$ be a time after Susie is told that it is Monday, but before the coin is flipped. Let Ch_1 be the objective chance function at $t = 1$, and let Ch_2 be the objective chance function at $t = 2$. Because the coin is fair, the chance of heads at times $t = 1$ and $t = 2$ is $\frac{1}{2}$. That is, $Ch_1(H) = Ch_2(H) = \frac{1}{2}$.

Let Cr_1 be the credence function which Susie ought to have at $t = 1$, and let Cr_2 be the credence function which Susie ought to have at $t = 2$. A formal connection between these two credence functions, endorsed by Elga (2000), Lewis (2001), and many others, is that

$$Cr_2(H) = Cr_1(H \mid Mo) \tag{1}$$

In other words, Susie's credence function at time $t = 2$ is just her credence function at time $t = 1$ updated with the only information she received in the interim: the information that it

is Monday.

Since Susie knows the details of the experiment, she knows that H_t is impossible: if the coin lands heads, then she will not be woken on Tuesday. So

$$Cr_1(H_t) = 0 \tag{2}$$

In addition, the Principal Principle can be used to derive the following two equations:

$$Cr_1(H) = \frac{1}{2} \tag{3}$$

and

$$Cr_2(H) = \frac{1}{2} \tag{4}$$

The derivations of (3) and (4), from the Principal Principle, are fully rigorous: see the appendix for details. But mathematical rigor aside, it is pretty intuitive that these two equations follow from the Principal Principle. The quick-and-dirty way of describing the Principal Principle goes like this: in order to be rational, agents must set their credences equal to the known chances. And that is what equations (3) and (4) do. They follow from two facts: (i) at times $t = 1$ and $t = 2$, the chance of the coin landing heads is $\frac{1}{2}$, and (ii) at times $t = 1$ and $t = 2$, Susie knows that those are the chances at those times.²

Now for the principal problem: as proved in the appendix, equations (1), (2), (3), and (4) jointly imply that $Cr_1(Mo) = 1$. In other words, those four equations jointly imply that in order to be rational, Susie must be completely confident that it is Monday at time $t = 1$. But her Monday awakening is just like her Tuesday awakening. And at time $t = 1$, she has

²To be clear: when Susie wakes up at time $t = 1$, she might not know that the *current* chance of the coin landing heads is $\frac{1}{2}$. In other words, she might not know the following centered proposition: right now, $\frac{1}{2}$ is the chance of the coin landing heads. But since Susie knows all the details of the experiment, she *always* knows that at time $t = 1$, the chance of the coin landing heads is $\frac{1}{2}$. She always knows the uncentered proposition that $Ch_1(H) = \frac{1}{2}$. Similarly, Susie *always* knows that at time $t = 2$, the chance of the coin landing heads is $\frac{1}{2}$. So she always knows the uncentered proposition that $Ch_2(H) = \frac{1}{2}$. And as shown in the appendix, this suffices to derive equations (3) and (4).

not yet been told that it is Monday. So at time $t = 1$, it is obviously irrational for Susie to be completely confident that it is Monday. Thus, when applied to centered propositions, the Principal Principle forces rational agents to have irrational credences.

2.3 Responses and Replies

In this section, I discuss three possible responses to the principal problem, each different from the response I will ultimately endorse. As shall become clear, the none of the three succeed.

The first response denies equation (2): it denies that Susie's credences ought to be such that $Cr_1(H_t) = 0$. This response, however, is implausible. Since Susie knows all the details of the experiment at time $t = 1$, she knows it is impossible for the coin to land heads and for it to be Tuesday. In other words, Susie's credence in H_t is zero at time $t = 1$; that is, $Cr_1(H_t) = 0$.

The second response denies equation (1): it denies that Susie's credences ought to be such that $Cr_2(H) = Cr_1(H | Mo)$. This response strikes me as plausible: standard Bayesian conditionalization probably does not hold in situations involving centered propositions. But ultimately, this response does not rescue the Principal Principle from the principal problem. As I show in the appendix, equation (1) is not necessary to derive the conclusion that at time $t = 1$, Susie must be completely confident that it is Monday. In fact, equation (4) is not necessary either. The same conclusion follows from equations (2), (3), and

$$Cr_1(H | Mo) = \frac{1}{2} \tag{5}$$

For two reasons, we ought to accept (5). First, (5) is intuitively plausible. At time $t = 1$, Susie knows that conditional on it being Monday, the chance of the coin landing heads is $\frac{1}{2}$, since she knows that the coin is fair and that the coin has not yet been flipped. So at $t = 1$, her credence in the coin landing heads – conditional on it being Monday – ought to

be $\frac{1}{2}$ as well: she ought to set her credences equal to the chances. And that is what (5) says.

Second, (5) follows from standard rules for updating on centered propositions. For instance, it follows from Meacham’s ‘compartmentalized conditionalization’ rule (2008, p. 257). So (5) is not just intuitive. It follows from standard theories of updating on propositions like “It is Monday”.

The third response to the principal problem can be extracted from a claim made by Lewis (2001): *Mo*—the proposition that it is Monday—is inadmissible. As discussed in the appendix, the derivation of (4) from the Principal Principle assumes the admissibility of *Mo*. So if *Mo* is inadmissible, then the Principal Principle does not imply (4).

But as mentioned above, (4)—like (1)—is not necessary to derive the conclusion that at time $t = 1$, Susie must be completely confident that it is Monday. The same conclusion follows from equations (2), (3), and (5). So the principal problem persists, even without (4).³

3 Centering the Principal Principle

The Principal Principle gives us a window into the nature of chance: it tells us something about what chance is like. So any perplexing implications of the Principal Principle are telling us something about chance. In particular, the perplexing implications of Susie’s experiment are a sign that we should reevaluate our understanding of the chances of propositions like *H*.

³There is another reason to reject this response: Lewis’s argument for the inadmissibility of *Mo* is not particularly persuasive. According to Lewis, *Mo* is inadmissible because (i) it is “about the future” (2000, p. 175), and (ii) *Mo* changes Susie’s credence in *H*, the proposition that the coin lands head. But it is not clear that *Mo* is ‘about’ the future in any sense relevant to Susie’s experiment. *Mo* seems to be ‘about’ the present. And even if *Mo* is ‘about’ the future in some way, it does not follow that *Mo* must be inadmissible simply because it changes Susie’s credence in *H*. Plenty of propositions are about the future, and change agents’ credences in other propositions, but are perfectly admissible. For example, suppose Billy is told that two fair coins will be tossed in two days. Because of that, his credence in the proposition that at least one coin comes up heads – call this proposition *C* – is $\frac{3}{4}$, as that is the current chance of *C*. The next day, Billy learns the proposition *L*: one of the coins has been lost, and so only one coin will be flipped. This proposition is ‘about’ the future, in the sense that it tells Billy something about the future coin-flipping event. And it changes Billy’s credence in *C* to $\frac{1}{2}$, since it changes the chance of *C* to $\frac{1}{2}$ as well. But *L* is not inadmissible. Billy does everything correctly when he uses the Principal Principle to adjust his credence in *C* to the new chance of *C*.

So in this section, I propose a principle which has the exact same mathematical form as the Principal Principle, but which has radically different content. It allows for centered chances; that is, it allows centered propositions to be objectively chancy.

Centered chances might sound bizarre. What could it mean to say that a centered proposition has an objective chance? How could a centered proposition like “It is Monday” have a chance of obtaining? What sort of thing could centered chance be?

To answer these questions about centered chance, it helps to recall that similar questions have been raised about uncentered chance. What could it mean to say that an uncentered proposition has an objective chance? How could an uncentered proposition like “The atom decays” have a chance of obtaining? What sort of thing could uncentered chance be?

One answer, which I very much like, invokes the Principal Principle: uncentered chances are those things which constrain rational credences in the way that the Principal Principle describes. Lewis subscribes to something like this when he says that the Principal Principle “captures all we know about chance” (1980, p. 266). Though this strikes me as an exaggeration, I find it a helpful one. The chance of an uncentered proposition A is that which, via the Principal Principle, constrains rational credence in A .

I propose a similar answer for questions about the nature of centered chance: centered chances are those things which constrain rational credences in the way that the following principle—call it the ‘Centered Principal Principle’—describes. Let A be a proposition, centered or uncentered. Let t be a time. Let Ch be a chance function relative to a particular agent B : note that unlike the chance function in the original Principal Principle, this chance function is relativized to agents. Suppose that Ch is defined, at time t , over an algebra of which A is a member. Let x be a real number in the unit interval. And let E be any proposition which is consistent with $Ch(A) = x$, and which is admissible at time t . Then B ’s initial credence function Cr ought to satisfy the following equation.

$$Cr(A \mid E \ \& \ Ch(A) = x) = x$$

Though the Centered Principal Principle takes the form of the Principal Principle, they are extremely different. In the Centered Principal Principle, the chance function Ch is defined relative to agents. And because of that, Ch may differ from one agent to the next.⁴

This is a feature of the Centered Principal Principle, not a bug. For the same centered proposition can have different chances for different agents. In everyday circumstances, the chance of the proposition “I am Beth”—relative to Beth—is 1: Beth is Beth with unit chance.⁵ In everyday circumstances, the chance of this proposition—relative to Billy—is 0: there is no chance that Billy is Beth.⁶

This relativity of centered chance to agents is akin to the relativity of chance—centered or uncentered—to times. The chance of a proposition can differ from one time to another. Times, of course, can be centers of centered propositions. Therefore, chances can vary along one kind of center: namely, the kind corresponding to times. The relativity of centered chance to agents is just variation along another kind of center: namely, the kind corresponding to agents. So the fact that centered chances can vary from agent to agent is quite similar to the fact that chances can vary from time to time.

Another difference between the Centered Principal Principle and the Principal Principle concerns the possible values which $Ch(A)$ can take for certain A . Recall that in Section 2.2, I assumed that the chance of the coin landing heads was $\frac{1}{2}$: I assumed that $Ch_1(H) = Ch_2(H) = \frac{1}{2}$. I made this assumption because $\frac{1}{2}$ is the *uncentered* chance of a fair coin landing heads. But H is, of course, a centered proposition. So for Susie, the chance of the centered proposition H can differ from the chance of the uncentered proposition—call

⁴In Section 4, I discuss chance functions which are relativized to communities of agents, not just individual agents. So the chance functions invoked in the Centered Principal Principle can also be relativized to entire communities.

⁵I include the qualifier ‘in everyday circumstances’ because there may be unusual circumstances in which the centered chance of “I am Beth”, relative to Beth, is less than 1. Suppose Beth briefly suffers amnesia, and cannot remember whether she is Beth or Bailey. In a circumstance like that, “I am Beth” may, relative to Beth, have a centered chance of $\frac{3}{4}$.

⁶That is the extent of the possible disagreement between chance functions, however. Chance functions for different agents do not disagree on uncentered propositions: in other words, for any uncentered proposition P , and for any agents B and B' , the chance of P for B equals the chance of P for B' .

it ‘ U ’—that the coin lands heads.⁷ In other words, at times $t = 1$ and $t = 2$, the centered chances of the coin landing heads – namely, $Ch_1(H)$ and $Ch_2(H)$ – may well differ from the uncentered chances of the coin landing heads – namely, $Ch_1(U)$ and $Ch_2(U)$. The Centered Principal Principle allows for this possibility.

So what, exactly, is the centered chance of H (relative to Susie)? What are the values of $Ch_1(H)$ and $Ch_2(H)$? There are at least two different methods for determining the chances of centered propositions. In the next section, I discuss one of those methods: the centered chance of a proposition is the chance assigned to that proposition by the world’s best deductive system. For now, however, let me focus on another: centered chances can be determined from rational centered credences, using the Centered Principal Principle. In particular, to figure out the centered chances, do the following. First, figure out—by whatever means are available—the rational centered credences. Second, use the Centered Principal Principle to reverse-engineer what the centered chances must be, in order for those centered credences to indeed be rational.

So for example, to figure out the centered chance of H at a time, relative to Susie, do the following. First, figure out the centered credence which Susie ought to have in H at that time. I am a thirder: so following Elga (2000), Horgan (2004), Weintraub (2004), Weatherson (2013), and others, I think that at time $t = 1$, the rational credence for Susie to have in H is $\frac{1}{3}$, and at time $t = 2$, the rational credence for Susie to have in H is $\frac{1}{2}$. In other words, Susie’s credences ought to be such that $Cr_1(H) = \frac{1}{3}$ and $Cr_2(H) = \frac{1}{2}$. Second, use the Centered Principal Principle to determine the centered chances of H at those times, relative to Susie. The Centered Principal Principle suggests that (i) at time $t = 1$, the chance of H (relative to Susie) is the credence in H which Susie ought to have, and (ii) at time $t = 2$,

⁷Because of this, I am guilty of an overly-hasty inference. In Section 2, I implicitly inferred that the centered chance of the coin landing heads is $\frac{1}{2}$ from the fact that the uncentered chance of the coin landing heads is $\frac{1}{2}$; that is, I implicitly inferred that $Ch_1(H) = \frac{1}{2}$ from the fact that $Ch_1(U) = \frac{1}{2}$. This overly-hasty inference is ubiquitous in the literature on Elga’s ‘Sleeping Beauty’ case: everyone assumes that in Elga’s case, the objective chance of the coin landing heads is $\frac{1}{2}$. And it is this overly-hasty inference which leads to the principal problem. Presumably, this inference is so common in the literature because the distinction between centered chance and uncentered chance has been overlooked.

the chance of H (relative to Susie) is the credence in H which Susie ought to have. In other words, $Ch_1(H) = \frac{1}{3}$ and $Ch_2(H) = \frac{1}{2}$.

This solves the principal problem. For all the reasons given above, suppose that relative to Susie, $Ch_1(H) = \frac{1}{3}$. Then (3)—the proposition that $Cr_1(H) = \frac{1}{2}$ —does not follow from the Centered Principal Principle. For according to the Centered Principal Principle, Susie’s initial credence function Cr ought to be such that $Cr(H \mid \mathcal{H}_1 \ \& \ Ch_1(H) = \frac{1}{3}) = \frac{1}{3}$, where \mathcal{H}_1 describes the complete history of the world up to time $t = 1$. Since Susie knows both \mathcal{H}_1 and $Ch_1(H) = \frac{1}{3}$ at time $t = 1$, it follows⁸ that $Cr_1(H) = \frac{1}{3}$.

The chance of H at time $t = 1$ (relative to Susie) differs from the chance usually associated with a fair coin landing heads. But that is fine. The lesson of Susie’s experiment is that centered chances can depart from uncentered chances. When the experimenters tell Susie that the coin is fair, they are telling her that $\frac{1}{2}$ is the *uncentered* chance of the coin landing heads: they are telling her the chance of U . That is compatible with the *centered* chance of the coin landing heads, at time $t = 1$, being $\frac{1}{3}$. For H and U are different propositions: roughly put, H is the disjunction of H_m and H_t , and U is the disjunction of all centered possible worlds in which the coin lands heads.⁹

So what sorts of things could centered chances be? They are the sorts of things that, when known, constrain rational centered credences. They are the sorts of things that relate to centered credences in the manner characterized by the Centered Principal Principle. So the proper response to concerns about centered chance parallels Lewis’s response to similar concerns about uncentered chance. According to Lewis, (uncentered) chance is the sort of thing that constrains rational (uncentered) credence in the way that the Principal Principle describes. I say similarly for centered chance. *Centered* chance is the sort of thing that constrains rational *centered* credence in the way that the *Centered* Principal Principle

⁸This follows from two facts: $Cr(Ch_1(H) = \frac{1}{3}) = 1$, since at the initial time, Susie knows the details of the experiment; and $Cr_1(H) = Cr(H \mid \mathcal{H}_1)$, since Susie’s credence function at time $t = 1$ is her initial credence function conditional on the history of the world up to that time.

⁹More precisely, H is $\{\langle w, x \rangle \mid w \text{ is a world in which the coin lands heads, and } x \text{ is either time } t = 1 \text{ or time } t = 2\}$, and U is $\{\langle w, t \rangle \mid w \text{ is a world in which the coin lands heads, and } t \text{ is a time}\}$.

describes.

Here is another way to think about it: centered chance is that which provides the objective basis for centered credences, much like uncentered chance is that which provides the objective basis for uncentered credences. Centered chances are those worldly items which guide rational credences about what time it is, who we are, and so on. Centered chances explain why some centered credences are rational and other centered credences are not.

4 Centered Chance and Physical Law

I think that the remarks in the previous section are sufficient to explicate the notion of centered chance. Nevertheless, one might still want something more: one might want an analysis. So in this section, I analyze centered chance using the best system account of laws; the analysis is similar to the best system analyses of uncentered chance offered by Lewis (1994) and Loewer (2004). Then I motivate my analysis by appealing to contemporary physics: considerations from cosmology, and from Everettian quantum mechanics, suggest that our best physical theory of the world may invoke centered chances.

By way of preparation, recall the best system account of lawhood. Laws, according to the best system account, are theorems of those deductive systems that best balance a variety of theoretical virtues: traditionally, these include simplicity, strength, and fit.¹⁰ In slogan form: to be a law is to be part of the best summary of the world. And uncentered chances, on this account, are propositions which (i) assign probabilities to uncentered propositions, and which (ii) the best deductive systems imply (Lewis, 1994, p. 480; Loewer, 2004, pp. 1118-1119).

I propose that we take centered chances to be the same sorts of things: centered chances are propositions which (i) assign probabilities to centered propositions, and which (ii) the

¹⁰See Loewer (2004, p. 1119) for a description of these three virtues. See Elga (2004) for discussion of a problem concerning fit, and a candidate solution.

best deductive systems imply. In other words, centered chances fall out of the laws. They derive from the best summary of the world. Call this the ‘best system analysis’ of centered chance.

As an example of a world with a best deductive system like that, consider a world \mathfrak{w} in which something like Susie’s experiment happens to everyone, and it does so multiple times. Each Sunday night, everyone falls asleep. On Monday, everyone wakes up. A short time later, everyone learns that it is Monday: perhaps the message “It is Monday” flashes across the sky. Then everyone falls back asleep. Later that evening, as a result of a purely natural mechanism—a mechanism outside of everyone’s control—a coin is flipped. If the coin lands heads, everyone wakes up on Wednesday and goes about their lives as usual until Sunday, whereupon the process repeats. If the coin lands tails, an amnestic gas is released across the world. Everyone breathes it in, has their memory of the Monday awakening erased, and eventually wakes up on Tuesday. Then everyone falls back asleep, wakes up on Wednesday with their memories restored, and goes about their lives as usual until Sunday, whereupon the process repeats. In other words, everyone in \mathfrak{w} undergoes a version of Susie’s experiment, except that there are no experimenters. Everyone experiences regular, weekly, *de se* uncertainty.

The best system at \mathfrak{w} posits a centered chance. For best systems are summaries: they concisely and informatively summarize all the non-modal facts. Facts about the relative frequency of it being Monday—that is, facts about the frequency with which the centered proposition “It is Monday” obtains—are non-modal. In particular, there is a non-modal fact about the frequency with which, once everyone awakens, it turns out to be Monday: that happens approximately two-thirds of the time. And there is a non-modal fact about the frequency with which, once everyone awakens, it turns out to be Tuesday: that happens approximately one-third of the time. The best summaries of these frequency facts invoke centered chances. The centered chance of it being Monday once everyone awakens – relative to everyone – is $\frac{2}{3}$, and the centered chance of it being Tuesday once everyone awakens –

relative to everyone – is $\frac{1}{3}$. So the best system at \mathfrak{w} posits centered chances, in order to summarize all that happens.

In the previous section, I outlined a method for determining centered chances which relies on centered credences. The best system analysis provides an alternative method for determining centered chances: determine the best deductive system of the world, and extract the centered chances from that. As the example of \mathfrak{w} shows, this method relies crucially on frequencies. The best system summarizes the frequencies with which centered propositions—like the proposition “It is Monday”—obtain. So the chances of centered propositions are whatever chances, when assigned to those propositions, provide the best summary of the relevant frequencies.

This method, for determining centered chances, provides a new argument for the view that at $t = 1$ —that is, when Susie wakes up on Monday but before she is told that it is Monday—Susie’s credence in H ought to be $\frac{1}{3}$. Suppose Susie’s experiment is repeated many, many times. Let $t = 1$. Then consider all the events—past, present, and future—in which Susie wakes up, but Susie is unaware of whether it is Monday or Tuesday. In approximately two-thirds of those events, it turns out to be Monday. So according to the best summary of that frequency, at time $t = 1$ the centered chance of it being Monday is $\frac{2}{3}$ (relative to Susie). In other words, according to the best system, $Ch_1(Mo) = \frac{2}{3}$ (relative to Susie). For similar reasons, the best system implies that $Ch_1(H_t) = 0$ and $Ch_1(H \mid Mo) = \frac{1}{2}$.¹¹ As a simple calculation shows, these three equations imply that relative to Susie, $Ch_1(H) = \frac{1}{3}$. And so according to the Centered Principal Principle, Susie’s initial credence function Cr ought to be such that $Cr(H \mid \mathcal{H}_1 \ \& \ Ch_1(H) = \frac{1}{3}) = \frac{1}{3}$. As shown in Section 3, it follows

¹¹Here is why. Let $t = 1$, and consider all the events in which Susie wakes up, but Susie is unaware of whether it is Monday or Tuesday. In each of those events, it is *not* the case that (i) it is Tuesday, and yet (ii) the coin lands heads. So according to the best summary of that null frequency, the centered chance of it being Tuesday and the coin landing heads—at time $t = 1$, relative to Susie—is 0. That is, relative to Susie, $Ch_1(H_t) = 0$. Now consider all the events in which (i) Susie wakes up, (ii) Susie is unaware of whether it is Monday or Tuesday, but (iii) it is, as a matter of fact, Monday. In approximately half of those events, the coin ends up landing heads. So according to the best summary of that frequency, the centered chance of the coin landing heads *given that it is Monday*—at time $t = 1$, relative to Susie—is $\frac{1}{2}$. In other words, $Ch_1(H \mid Mo) = \frac{1}{2}$.

that $Cr_1(H) = \frac{1}{3}$; that is, Susie ought to have credence $\frac{1}{3}$ in the coin landing heads.¹²

One might object that to summarize what happens at worlds like \mathfrak{w} , it suffices to summarize all the *de dicto* facts. So the best systems at worlds like \mathfrak{w} do not invoke centered chances. But that, I think, is implausible. Imagine a community of scientists in \mathfrak{w} , trying to come up with the best theory of their world. These scientists would want to capture the bizarre connection between the natural coin-flipping mechanism and their experiences of waking and sleeping. Those scientists would want to describe the frequencies with which propositions like “It is Monday” obtain. To do so, they would posit a centered chance: the chance that upon awakening, it is Monday. So their best summary of \mathfrak{w} would summarize *de se* facts as well as *de dicto* facts.

Alternatively, one might object that even if centered chances do feature in some best systems, that only happens at bizarre worlds like \mathfrak{w} . Centered chances are utterly absent, one might claim, from the best system at our world. But as I shall argue, that is doubtful. In the remainder of this section, I give two reasons for thinking that the best system at the *actual* world invokes centered chances. One reason derives from contemporary cosmology, and the other derives from the Everett interpretation of quantum mechanics.

According to contemporary cosmology, in large or infinite universes with laws like ours, it is virtually guaranteed that the data we actually observe—the patterns of incoming light, the distribution of observable matter, and so on—occur somewhere or other in spacetime. And it is virtually guaranteed that many observers, at different spatiotemporal locations, will see such data. Some physicists have argued that because of this, we need more than just accounts of the probabilities that various physical states will occur—or that various data will

¹²This argument for being a thirder is somewhat similar to, yet importantly distinct from, an argument given by Elga (2000). Elga’s argument purports to derive the rational credences directly from the frequencies: at time $t = 1$, Susie ought to have credence $\frac{1}{3}$ in the coin landing heads because in the long run, approximately one-third of her wakings would be Heads-wakings (2000, pp. 143-144). My argument for being a thirder is different, insofar as it relies on an important intermediary step. In my argument, the rational credence in H derives from (i) the centered chance of H , and (ii) the Centered Principal Principle. The centered chance of H is determined by the best deductive system. So the rational credence in H is determined by more than just the frequencies. The rational credence in H is determined by (i) the best *summary* of those frequencies, and (ii) a principle which links those summaries to the credences which agents ought to have.

be observed—in order to successfully test our theories (Srednicki & Hartle, 2010). We need accounts of the probability that *we*, rather than observers elsewhere, will observe that data. In other words, we need centered chances to account for the way in which *our* data confirms or disconfirms physical theories.

Now consider the Everett interpretation of quantum mechanics. According to some versions of the Everett interpretation, observers regularly experience *de se* uncertainty (Sebens & Carroll, 2018; Vaidman, 2014). For example, suppose Alice does an experiment to determine the spin properties of an electron. After the experiment concludes, but before Alice looks at the outcome, she is uncertain about whether the electron was found to have one spin property or another. A rough description of the situation goes like this: at the moment of the experiment, the world splits into two ‘branches’—one in which the electron has one spin property, and one in which the electron has the other spin property—and Alice is uncertain of which branch she is on (Sebens & Carroll, 2018, p. 33). There are two copies of Alice, one on each branch, and Alice is unsure of which is her. So Alice has a centered credence in the proposition that she is on one branch, and a centered credence in the proposition that she is on the other branch.

Centered chances provide an objective, physical basis for Alice’s centered credences.¹³ Let us suppose that Alice ought to have credence $\frac{1}{2}$ in being on one branch, and credence $\frac{1}{2}$ in being on the other.¹⁴ Then plausibly, the centered chance of Alice being on one branch is $\frac{1}{2}$, as is the centered chance of Alice being on the other branch. In fact, these centered chances can be derived from the quantum state of the electron, given a particular posit about how quantum states determine centered chances.¹⁵ That posit is part of the best system of an Everettian world, since that posit provides a simple, informative summary of

¹³This is compatible with Alice’s centered credences being justified in other ways. It is compatible, for example, with Alice’s centered credences being justified by principles of rationality, such as the epistemic separability principle endorsed by Sebens and Carroll (2018, p. 40).

¹⁴Assume that the electron’s spin state before measurement is $\frac{1}{\sqrt{2}}|\uparrow_x\rangle + \frac{1}{\sqrt{2}}|\downarrow_x\rangle$, where $|\uparrow_x\rangle$ represents the electron being in the ‘x-spin up’ state and $|\downarrow_x\rangle$ represents the electron being in the ‘x-spin down’ state.

¹⁵The posit is basically a slightly reworded version of the Born rule: the centered chance (for the experimenters) of ending up on a branch corresponding to eigenvalue a of observable \hat{A} , given a system prepared in state $|\psi\rangle$, is $|\langle a|\psi\rangle|^2$, where $|a\rangle$ is the eigenvector corresponding to a .

where we generally find ourselves in a world like that. So if the actual world is Everettian, then plausibly, the best system of the actual world posits centered chances.

Advocates of the best system account of lawhood often explain their view by appealing to an imagined conversation with God (Albert, 2015, p. 23). So in closing, let me do likewise for the best system analysis of centered chance. You ask God to tell you about the world. God begins to recite a long litany of facts: this particle is over here, that one is over there, and so on. Pressed for time, you ask God if there is some simple yet highly informative summary of what the world is like. In granting your request, God gives you the laws. For laws are simple yet highly informative summaries.

Then you notice that the laws allow for worlds in which many physical subsystems, all near-duplicates of one another, look a whole lot like you. You notice, in other words, that the laws allow for situations in which you could be this physical system over here, or that physical system over there, or that other one, and so on. Perhaps this is because God has given you the laws of contemporary cosmology, or the laws of Everettian quantum mechanics, or the laws of the bizarre world \mathfrak{w} .

So you say to God: “I want to be able to *check* these laws. I want to run experiments on them, to see whether or not they are true. But to do that, I need to know which physical subsystem of the world is me. Or at least, I need to know *something* about where I am in the world. Otherwise, I won’t know whether the data I collect is from this part of the world, or that part, and so on. And so I won’t be able to empirically confirm the laws”.

God says: “Okay. But since you’re in a rush, I won’t bother specifying exactly which subsystem you are at each moment in time. Instead, I’ll just give you some chancy rules to help guide your guesses as to where you might be”.

That, according to the view presented in this section, is centered chance. Centered chances are the things that God would give you, to help you determine where you are.

5 Conclusion

Susie's experiment reveals a problem with our best theories of the link between credence and chance. In particular, when applied to centered propositions, the Principal Principle implies that Susie must be irrationally confident in it being Monday.

The solution: posit centered chances, and require centered credences to equal those centered chances (when the centered chances are known). In other words, adopt the Centered Principal Principle. This solution might seem costly, since centered chances might seem strange. But they can be explicated using the Centered Principal Principle, and they can be analyzed using the best system account of lawhood – just as uncentered chances can be explicated using the Principal Principle, and just as uncentered chances can be analyzed using the best system account. Because of that, and because they help us avoid the principal problem, centered chances are worth positing.

6 Appendix

In this appendix, I derive equations

$$Cr_1(H) = \frac{1}{2} \tag{3}$$

and

$$Cr_2(H) = \frac{1}{2} \tag{4}$$

from the Principal Principle, the New Principle, and the General Recipe. Then I show how those two equations, together with equations

$$Cr_2(H) = Cr_1(H \mid Mo) \tag{1}$$

and

$$Cr_1(H_t) = 0 \tag{2}$$

imply that, when Susie wakes up on Monday but before she is told that it is Monday, she ought to be completely confident that it is Monday. Finally, I point out that the same conclusion follows from just equations (2), (3), and

$$Cr_1(H \mid Mo) = \frac{1}{2} \tag{5}$$

Here is the derivation of (3) and (4) from the Principal Principle. To start, recall that the Principal Principle concerns agents' *initial* credence functions – their credence functions at some earlier time. So let Cr be Susie's initial credence function. Following Elga (2000) and Lewis (2001), we may stipulate that Cr_1 is equal to Susie's initial credence function Cr conditionalized on the proposition \mathcal{H}_1 , where \mathcal{H}_1 expresses the complete history of the world up to time $t = 1$. That is, for all propositions A ,

$$Cr_1(A) = Cr(A \mid \mathcal{H}_1) \tag{6}$$

Similarly, Cr_2 is equal to Cr conditionalized on the proposition \mathcal{H}_2 , where \mathcal{H}_2 expresses the complete history of the world up to time $t = 2$. That is, for all propositions A ,

$$Cr_2(A) = Cr(A \mid \mathcal{H}_2) \tag{7}$$

According to the Principal Principle,

$$Cr \left(H \mid \mathcal{H}_1 \ \& \ Ch_1(H) = \frac{1}{2} \right) = \frac{1}{2} \tag{8}$$

and

$$Cr \left(H \mid \mathcal{H}_2 \ \& \ Ch_2(H) = \frac{1}{2} \right) = \frac{1}{2} \tag{9}$$

This assumes, of course, that \mathcal{H}_1 and \mathcal{H}_2 are admissible. But that is quite reasonable. Propositions about the history of the world are paradigmatic examples of admissible propositions: Lewis, for example, says as much (1980, p. 272). So this is a reasonable assumption to adopt.

We may suppose that Susie knows \mathcal{H}_1 at time $t = 1$, and Susie knows \mathcal{H}_2 at time $t = 2$.¹⁶ Since Susie is told that the coin is fair, she knows that $Ch_1(H) = \frac{1}{2}$ and that $Ch_2(H) = \frac{1}{2}$.¹⁷ Therefore, equations (6) and (8) jointly imply equation (3).¹⁸ Similarly, equations (7) and (9) jointly imply equation (4).¹⁹

I now derive equations (3) and (4) from a different version of the Principal Principle, which was proposed by Hall (1994, p. 511) and Lewis (1994, p. 487). Let A be a proposition. Let t be a time. Let Ch be a chance function defined, at time t , over an algebra of which A is a member. Let \mathcal{H}_t be the proposition that completely characterizes the history of the world up to time t . Let T be the proposition expressing the complete theory of chance at the actual world.²⁰ Then a rational agent's initial credence function Cr ought to satisfy the following equation.

$$Cr(A \mid \mathcal{H}_t \ \& \ T) = Ch(A \mid T)$$

Call this the 'New Principle'.

To streamline the derivation of (3) and (4) from the New Principle, suppose that the

¹⁶This follows from two stipulations about the case. First, Susie knows the complete history of the world up to the time she is told the details of the experiment; perhaps this is a world where that history is not very complicated. Second, nothing happens between time $t = 1$ and time $t = 2$, apart from Susie being told that it is Monday.

¹⁷For a detailed explanation of why Susie knows that $Ch_1(H) = \frac{1}{2}$, even at time $t = 1$, see footnote 2.

¹⁸To make the derivation fully rigorous, we must specify exactly when Cr is Susie's credence function. For simplicity, suppose that Cr is Susie's credence function right after being told all the details of Susie's experiment. Then $Cr(Ch_1(H) = \frac{1}{2}) = 1$, since Susie is told that the coin is fair. Given this, (8) reduces to $Cr(H \mid \mathcal{H}_1) = \frac{1}{2}$. And this, in conjunction with (6), implies (3).

¹⁹As mentioned in footnote 18, to make the derivation fully rigorous, stipulate that Cr is Susie's credence function right after Susie is told all the details of the experiment. Then $Cr(Ch_2(H) = \frac{1}{2}) = 1$, since Susie is told that the coin is fair. From this, (9) reduces to $Cr(H \mid \mathcal{H}_2) = \frac{1}{2}$. And this, in conjunction with (7), implies (4).

²⁰So T lists all the relevant history-to-chance conditionals (Lewis, 1994, p. 487). A history-to-chance conditional is a specification of what the chances are, given a complete history of the world. It is a conditional of the form "If the complete history of the world is thus-and-so, then the chances are such-and-such".

complete chance theory T only specifies the chances of the coin landing heads (no other chancy events happen in the world). So T is the proposition that the chance of the coin landing heads is $\frac{1}{2}$ and the chance of the coin landing tails is $\frac{1}{2}$. Therefore, $Ch_1(H | T) = \frac{1}{2}$ and $Ch_2(H | T) = \frac{1}{2}$. As before, we may suppose that Susie knows \mathcal{H}_1 at time $t = 1$, and Susie knows \mathcal{H}_2 at time $t = 2$. Since Susie was told all the details of the experiment, Susie knows the chance theory T . Therefore, $Cr_1(H | \mathcal{H}_1 \ \& \ T) = Cr_1(H)$ and $Cr_2(H | \mathcal{H}_2 \ \& \ T) = Cr_2(H)$. By the New Principle, $Cr_1(H | \mathcal{H}_1 \ \& \ T) = Ch_1(H | T)$ and $Cr_2(H | \mathcal{H}_2 \ \& \ T) = Ch_2(H | T)$. Therefore, $Cr_1(H) = Cr_2(H) = \frac{1}{2}$; that is, (3) and (4) follow.

Now consider a version of the Principal Principle due to Ismael (2008, p. 298). Let A and t be as before. For each complete theory of chance T , let $Ch_T(A)$ be the chance of A at t according to T , and let a_T be an agent's subjective assessment of the probability of T at t . Then in order for this agent to be rational, her credence function Cr at t ought to satisfy the following equation.

$$Cr(A) = \sum_T a_T Ch_T(A)$$

Call this the 'General Recipe'.

To streamline the derivation of (3) and (4) from the General Recipe, suppose once again that the complete chance theory T only specifies the chances of the coin landing heads. Let $Ch_{1,T}$ be the chances, according to T , at time $t = 1$. Let $Ch_{2,T}$ be the chances, according to T , at time $t = 2$. Then $Ch_{1,T}(H) = \frac{1}{2}$ and $Ch_{2,T}(H) = \frac{1}{2}$. Because Susie was told all the details of the experiment, her subjective assessment of the probability of T is 1, both at $t = 1$ and at $t = 2$. So her subjective assessments of the probabilities of all other chance theories are equal to 0. Substituting these values into the General Recipe yields $Cr_1(H) = Cr_2(H) = \frac{1}{2}$; that is, (3) and (4) follow.

Now let us see why equations (1), (2), (3), and (4) imply that $Cr_1(Mo) = 1$. To start, note that since the conditional credence in (1) is well-defined, it follows that $Cr_1(Mo) > 0$. Therefore,

$$\begin{aligned}
\frac{1}{2} &= Cr_2(H) \\
&= Cr_1(H \mid Mo) \\
&= \frac{Cr_1(H \& Mo)}{Cr_1(Mo)} \\
&= \frac{Cr_1((H_m \vee H_t) \& (H_m \vee T_m))}{Cr_1(Mo)} \\
&= \frac{Cr_1(H_m)}{Cr_1(Mo)} \\
&= \frac{Cr_1(H_m) + Cr_1(H_t)}{Cr_1(Mo)} \\
&= \frac{Cr_1(H_m \vee H_t)}{Cr_1(Mo)} \\
&= \frac{Cr_1(H)}{Cr_1(Mo)} \\
&= \frac{\frac{1}{2}}{Cr_1(Mo)}
\end{aligned}$$

where (4) yields the first line, (1) yields the second line, (2) yields the sixth line, and (3) yields the ninth line. Multiplying through by $2Cr_1(Mo)$ yields $Cr_1(Mo) = 1$. In other words, at time $t = 1$ —that is, before Susie is told that it is Monday—Susie must be completely confident that it is Monday.

To see that equations (2), (3), and (5) imply $Cr_1(Mo) = 1$, replace the first two lines in the above sequence of formulas by (5). The remainder of the derivation is exactly as shown above: (2), (3), and facts about probability functions are used to derive the equation $\frac{1}{2} = \frac{\frac{1}{2}}{Cr_1(Mo)}$, and multiplying through by $2Cr_1(Mo)$ yields $Cr_1(Mo) = 1$.

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