Reply to Stalnaker

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In 'Models and Reality', Robert Stalnaker responds to the tensions discerned in *Modal Logic as Metaphysics* between contingentism in modal metaphysics and the use of Kripke models in the semantics of modal logic. Stalnaker holds that 'the tensions in our intuitive conception of modal phenomena are real: both the contingency intuitions and the Kripke structures have features that need to be reconciled' (section 5), and that 'The contingency intuitions are not unassailable data' (section 7). His strategy for achieving reconciliation involves accepting 'the contingency intuitions' while arguing that 'we can use models to give a fully realistic interpretation of a modal language without giving a realistic interpretation of the models we are using' (section 1). The paper follows up the strategy of Stalnaker's earlier work (2010, 2012). Of his defence of contingentism and its compatibility with orthodox Kripke semantics, he writes (in section 1):

> that defense was not as explicit as it should have been about the status that I take Kripke models to have, and about the relation between these models and the reality that one is using them to model. This paper is an attempt to spell out in a little more detail what I take this relation to be.

It is easy enough to make out the broad outlines of Stalnaker's big picture, but much harder to bring it sharply into focus. Recent work by Peter Fritz and Jeremy Goodman has revealed some of the complexities and difficulties beneath the surface of Stalnaker's account (Fritz 2016; Fritz and Goodman 2016). Stalnaker's present paper does not fully resolve them. Since aspects of his account remain unclear, my comments will be tentative.

1. *Stalnaker's realism and instrumentalism in model-theoretic semantics*

Consider this passage from Stalnaker's earlier book (2012, p. 41):

> the merely possible individuals, and the points in logical space used in Kripke models as I am interpreting them, are like the spatial points in a relativist's model of spatial structure. The intended subject matter of our modal theory consists of the actual individuals, the (actual) properties and relation[s] that they might exemplify, and the (actual) higher-order properties and relations that might be exemplified by properties, relations, and propositions, as well as by individuals. About all these things, our theory can be resolutely realistic.

The reader may get the impression here that Stalnaker is contrasting the actual individuals in a Kripke model (the members of the domain of its actual world) with the merely possible individuals in the model (the non-members of that domain that are members of the domain of other worlds in the model): the merely possible individuals play an instrumental role, while the actual individuals themselves belong to the intended subject matter of the modal

theory. In my discussion of Stalnaker's criterion of representational significance, I quoted a similar passage from his slightly earlier article (Stalnaker 2010, p. 24; Williamson 2013, p. 189). However, in his response to my critical discussion of his earlier work, Stalnaker now insists that there is 'no reason' why actual individuals in the Kripke model have to represent themselves (section 6). Thus even the identities of the members of the domain of the actual world of the model become representationally insignificant; what matters is only which individuals they 'correspond to'. Is that what Stalnaker means by being 'resolutely realistic' about such aspects of the model?

Rather than worry about whether Stalnaker has modified his view, I will concentrate on his account in the present paper. He formulates his overall view of the model theory's role thus (section 3):

> The central claim of this paper is that one may intend a realistic interpretation of a *language*, but use nonrealistically interpreted models as aids in stating the compositional semantics for the language. That is, one may use a model as a representational device, interposed between the language being interpreted and the reality that is its intended subject matter. So there will be a correspondence between the language and the model theory, and a further correspondence between certain features of the model structure and the reality being modeled.

To specify *which* features of the favoured model structure are to correspond to features of the modal reality being modelled, Stalnaker associates each world in the model with a group of permutations of the worlds and individuals, which induce permutations of all the set-theoretic entities based on the model too. Then a set-theoretic feature of the model should

correspond to something real according to a world *w* in the model if and only if the feature is invariant under all the permutations associated with *w*. Since being real according to the actual world should coincide with being real (*tout court*), a set-theoretic feature of the model should correspond to something real if and only if the feature is invariant under all the permutations associated with the actual world of the model. Depending on the nature of the invariant set-theoretic feature, it will correspond to a real object, property, relation, proposition, or whatever.

To see how this might work, we can use a toy model. For simplicity, we may first consider a model of propositional modal logic, with worlds but no domains of individuals. For the set W of worlds, we take a set of just six ordered pairs: {<0, 0>, <0, 1>, <1, 0>, <1, 1>, <2, 0>, <2, 1>}. The designated actual world of the model is <0, 0>. As Stalnaker makes clear, there is no requirement for the 'worlds' of a model to be worlds in any serious informal sense. Let the permutation group associated with the world <*i*, *j*> comprise just those permutations of W that map <*i, j*> to itself (as Stalnaker's account requires) and leave the first component of each pair fixed. One can think of <*i*, 0> and <*i*, 1> as qualitatively identical twin worlds, or mirror images of each other. Thus the permutation group associated with the actual world <0, 0> contains only four permutations: the identity permutation, the permutation that just swaps <1, 0> with <1, 1>, the permutation that just swaps <2, 0> with <2, 1>, and the permutation that makes both swaps. Since all those permutations leave each of <0, 0> and <0, 1> fixed, both those worlds in the model should correspond to real worlds: in the model, <0, 1>, though actually real, does not actually obtain. Each other world in the model is shifted by some of the permutations, so it should not correspond to a real world.

If sets of worlds in the model correspond to anything real, they correspond to propositions. For instance, each of the sets {<1, 0>, <1, 1>} and {<2, 0>, <2, 1>} is invariant under all the actual world permutations, so it should correspond to a real proposition. By contrast, each of the sets {<1, 0>, <2, 0>} and {<1, 0>, <2, 1>} is shifted by some of the permutations, and so should not correspond to a real proposition. But *which* real proposition is the set {<1, 0>, <1, 1>} supposed to correspond to? Obviously, nothing has yet been done to determine that. We cannot say that it is the proposition true in the worlds <1, 0> and <1, 1> and false in all other worlds, because neither <1, 0> nor <1, 1> is supposed to correspond to a real world. The set {<1, 0>, <1, 1>} is supposed to correspond to something real even though none of its members does. This is presumably the sort of phenomenon Stalnaker has in mind when he writes that 'we need not assume that the membership of the domain that is specified by a derived type is determined by the membership of the types from which it is derived' and that 'For the contingentist, the relational structure exhibited by the type theory will be ungrounded in a way that it would be grounded if the necessitist's metaphysical assumptions were true' (section 5). But then how *is* the correspondence to real propositions to be set up? Of course, in this particular case we could just *stipulate* a proposition for {<1, 0>, <1, 1>} to correspond to, preferably one that could be true in just two ways. But we cannot solve the problem for the whole infinite type-theoretic hierarchy by *ad hoc* stipulation alone. A more systematic approach is needed.

Similar issues arise once we move to models of quantified modal logic and give each world a domain of individuals. Consider another toy model, this one with just two worlds, *w* and *w**. By stipulation, the domain of individuals for *w* is {0, 1, 2, 3}; the domain of individuals for *w** is {4, 5, 6, 7}. Each world is associated with the permutations of

individuals that fix each individual in its domain and map 'even' individuals to 'even' ones and 'odd' individuals to 'odd' ones. Of course, this is just a convenient way of specifying the permutations; the *real* individuals are not supposed to be numbers, and the numbers representing them have no intrinsic significance. Let *w* be the actual world of the model. Thus the group of permutations of individuals associated with the actual world contains only four permutations: the identity permutation, the permutation that just swaps 4 with 6, the permutation that just swaps 5 with 7, and the permutation that makes both swaps. Since only individuals in the domain of the actual world, are invariant under all the associated permutations, only they should correspond to real individuals. Let F be the monadic property intension whose extension at *w* is {0, 2} and whose extension at *w** is {4, 6}. F is invariant under all four permutations, so it should correspond to a real property. But *which* real property should it correspond to? Even once we have associated the invariant individuals 0, 1, 2, and 3 in the model with real individuals, and the invariant worlds *w* and *w** with real worlds, we cannot say that F should correspond to the property possessed by just the real individuals corresponding to 0 and 2 at the real world corresponding to *w* and by just the real individuals corresponding to 4 and 6 at the real world corresponding to *w**, for neither 4 nor 6 is supposed to correspond to a real individual. The supposed correspondence between F and a real property is not determined by correspondences lower down the type-theoretic hierarchy.

An *ad hoc* stipulative approach rapidly becomes infeasible even for very low levels of the type-theoretic as the models increase towards more realistic levels of complexity, in both the number of worlds and the associated domains of individuals. But the problem goes deeper than that. For Stalnaker wants Kripke models to play a key role in mediating between the modal language and the realistic interpretation we eventually give it. He says

that his 'claim will be that we can use models to give a fully realistic interpretation of a modal language without giving a realistic interpretation of the models we are using' (section 1) and that 'one may intend a realistic interpretation of a *language*, but use nonrealistically interpreted models as aids in stating the compositional semantics for the language' (section 3). Moreover, he regards such a key role as well-motivated: he says that the model-theoretic representation 'has both a compelling intuitive motivation, and considerable success in clarifying the structure of modal discourse, so we should be reluctant to give it up, or even to treat it as a merely instrumental device' (section 2). Thus the model-theoretic intermediaries should be properly integrated into the final 'fully realistic interpretation' of the modal language.

The problem is less urgent for the interpretation of the non-logical primitive constants of the language, because they are normally interpreted by one-off stipulations. It is much more serious for the interpretation of the quantifiers and variables of the various types. It is that which is at stake when Stalnaker writes 'We interpret the model structure with a correspondence relation between the invariant elements of the domains of **individuals**, **properties**, **relations** and **propositions** and elements of the appropriate kind in the corresponding categories in our type theory' (section 5; the boldface italic expressions apply to appropriate set-theoretic ersatz items in the model, while the 'elements of the appropriate kind in the corresponding categories in our type theory' are the real individuals, properties, relations, and propositions). We are told very little about *which* correspondence it is. Of course, we have some prior informal understanding of what real individuals, properties, relations, and propositions are, and so of what it means to generalize over them, independently of the model theory. But if we fall back on that prior informal understanding to interpret the modal language, we are in effect *bypassing* the formal model theory,

contrary to Stalnaker's stated intention of using the model theory to mediate between the modal language and its realistically interpreted compositional semantics.

In his earlier book, Stalnaker has an appendix (B) discussing an approach to possible worlds semantics in terms of propositional functions, whose upshot is supposed to be that 'We can talk with a clear conscience, in the metalanguage, about a domain of possible individuals because we have shown how to reconcile that talk with more austere ontological commitments and how to do the compositional semantics in a way that assigns as values only properties, relations, and functions that actually exist, according to the metaphysics that is presupposed' (2012, pp. 147-8). One might hope that this alternative semantics would somehow bear on the problem. But on examination it does not help. In particular, it leaves the role of the worlds unreduced, and the problem already arises for worlds, as we saw above. Nor does it help explain how the intension F in the second toy model gets to correspond to a real property. Since Stalnaker does not talk about propositional functions in the present paper, I will say no more about them here.

The problems just discussed do not prevent Stalnaker from using the model theory as an algebraic device with which to specify a formal consequence relation for the modal language. But doing so falls far short of providing a compositional semantics for the modal language, since it does not provide its sentences with truth-conditions, even relative to specifications of the real objects, properties, relations, and propositions associated with its non-logical constants. For instance, if someone tells you which sentences are logical consequences of which sets of sentences, and that the 1-place predicate S corresponds to the property of being spherical, you cannot determine from that alone that $\Box \exists x\, Sx$ is true if and only if it is necessary that something is spherical; inequivalent readings of $\Box$ and

different domains for the quantifier may yield the same consequence relation. Of course, you may happen to know the truth-condition of the formula independently, from your informal understanding of the modal language, but that is not what Stalnaker needs, since it involves no mediating role for the model theory. Moreover, if the model theory is only an algebraic device for specifying a formal consequence relation for the modal language, no other link having been specified to the intended interpretation of the language, why should we be interested in that consequence relation? We have not yet even been told why we should expect it to preserve truth from premises to conclusion, let alone why we should expect its logical truths (the consequences of the empty set of premises) to be all and only the metaphysically universal formulas.

As far as I can see, Stalnaker has not yet elucidated an integral role for Kripke models to play in a realistically interpreted compositional semantics for a modal language.[1]

2. Comprehension principles for Stalnakerian modal logic

One of the important points of modal metaphysics on which Stalnaker and I agree, and on which we disagree with some other contributors to this volume, is that higher-order contingentism is a natural — though not automatic — generalization of first-order contingentism. As he says, 'If Hillary Clinton is a contingently existing object, then it seems

reasonable to think that the properties of being identical to Hillary, or being the daughter of Hillary are also contingent' (section 2). Stalnaker endorses such higher-order contingentism.

As *Modal Logic as Metaphysics* notes, higher-order contingentism involves restrictions on comprehension principles for higher-order modal logic. For instance, in the quoted passage, Stalnaker allows that there is the property of being the daughter of Hillary Clinton, which we may express in the higher-order modal language thus (reading the 1-place predicate *D* 'is the daughter of Hillary Clinton'.):

(1) $\exists X \,\Box\, \forall x \,(Xx \leftrightarrow Dx)$

But Stalnaker denies that there is *necessarily* such a property; in other words, he denies the necessitation of (1):

(N1) $\Box \exists X \,\Box\, \forall x \,(Xx \leftrightarrow Dx)$

In effect, Stalnaker rejects the natural modal comprehension principle Comp$_M$ for quantification into monadic predicate position, which provides necessitated instances such as (N1) (Williamson 2013, pp. 262-3).

Restricting Comp$_M$ has a cost: it obstructs normal second-order reasoning in modal contexts. For instance, suppose that in reasoning about counterfactual circumstances we assume the second-order generalization $\forall X \,\Phi(X)$. We may wish to infer the instance $\Phi(D)$;

but we have no general right to do that, given the failure of (N1); (1) is insufficient, since we have it only for the actual circumstances. Of course, first-order contingentism faces a similar problem. For instance, suppose that in reasoning about counterfactual circumstances we assume the first-order generalization $\forall x\, \phi(x)$. We may wish to infer the instance $\phi(d)$; but we have no general right to do that, given the failure of $\Box \exists x\, d{=}x$ under first-order contingentism; $\exists x\, d{=}x$ is insufficient, since we have it only for the actual circumstances. What my book emphasizes is that the failures of comprehension undermine the free use of the kind of reasoning that provides the most powerful reason for introducing higher-order logic in the first place (Williamson 2013, pp. 282-8). I now elaborate the point by showing how much higher-order modal logic we in effect assume when applying non-modal mathematics in scientific reasoning about phase spaces of possible states of a physical system (Williamson 2016b and 2016c).

In the most natural formalization of reasoning about phase spaces, specific states are treated like worlds, and sets of such states like propositions, so generalizing about sets of states corresponds to quantifying into sentence position. Stalnaker is a contingentist about propositions too, so similar issues arise. For instance, he allows that there is the proposition that Hillary Clinton is running for President in 2016, which we may express using quantification into sentence position thus (reading P 'Hillary Clinton is running for President in 2016'):

(2)　　　$\exists X\, \Box(X \leftrightarrow P)$

But Stalnaker denies that there is *necessarily* such a proposition; in other words, he denies

the necessitation of (2):

(N2) $\Box \exists X \Box (X \leftrightarrow P)$

In effect, Stalnaker rejects the natural modal comprehension principle Comp$_P$ for

quantification into sentence position, which provides necessitated instances such as (N1)

(Williamson 2013, p. 290). But the obvious semantics for interpreting a modal language with

quantification into sentence position over phase spaces verifies principles generalizing (2)

and (N2).

It is not even clear whether Stalnaker's principles entitle him to instances of the

comprehension principles *without* the initial occurrence of the necessity operator. For

instance, he will presumably allow that there is the property of *not* being the daughter of

Hillary Clinton (permutation-invariance is preserved under negation). We might formalize

that thus:

(3) $\exists X \Box \forall x (Xx \leftrightarrow \neg Dx)$

For Stalnaker, such a property exists only contingently: in some possible circumstances,

there is no Hillary Clinton and no property of not being her daughter; there is just no

distinction between being her daughter and not being her daughter. By classical modal

reasoning that Stalnaker accepts, in those circumstances many things would have not been her daughter. By (3), they would in effect have had the property of not being her daughter — even though there would have been no such property for them to have had. In other words, a predication can be true with respect to circumstances in which the property designated by the predicate does not exist. But this involves Stalnaker in treating predications as asymmetrical between subject and predicate with respect to existential commitment, for his preferred first-order modal logic includes the principle that a predication *cannot* be true with respect to circumstances in which the object designated by the subject term does not exist (Stalnaker 1994; see Williamson 2013, pp. 183-8 for discussion). In brief, predications strictly imply the existence of the object but do not strictly imply the existence of the property. By contrast, an attractive feature of Stalnaker's approach in the present paper is the symmetry of treatment between objects, properties, relations, and propositions.

In this reply, I have raised several challenges to Stalnaker's account of the semantics, logic, and metaphysics of modal discourse. I hope that they will encourage him to develop a more explicit theory, so that the strengths and weaknesses of his version of contingentism can be properly assessed.

Note

1   For a closely related debate on how the model theory of modal logic connects to

modal metaphysics with reference to Williamson 2013 see deRosset 2016, Kment 2016, and

Williamson 2016a.

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