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What's Wrong with Zeno?

There was a time in my school years when I have learned about Achilles and Tortoise "paradox" originated from Zeno. It was then clear that the ancient Greeks were arguing about this problem but contemporary science has clarified the issue. Yet to my surprise the problem is still debated over and over, despite the fact there exist mathematical proofs. I feel like reminding myself why this is not a paradox beyond reasonable doubt. And really, I think the problem is trivial. It is described in Wikipedia as follows:

In the paradox of Achilles and the Tortoise, Achilles is in a footrace with the tortoise.

Achilles allows the tortoise a head start of 100 meters, for example. If we suppose that each racer starts running at some constant speed (one very fast and one very slow), then after some finite time, Achilles will have run 100 meters, bringing him to the tortoise's starting point. During this time, the tortoise has run a much shorter distance, say, 10 meters. It will then take Achilles some further time to run that distance, by which time the tortoise will have advanced farther; and then more time still to reach this third point, while the tortoise moves ahead. Thus, whenever Achilles reaches somewhere the tortoise has been, he still has farther to go. Therefore, because there are an infinite number of points Achilles must reach where the tortoise has already been, he can never overtake the tortoise.[9][10]

It was possible for the ancient Greeks to argue, because they did not even have a clear concept of motion and velocity – using only intuition.

It is enough to say that since then we have equations of motion which show otherwise. Consistently with experience we should stop thinking about paradoxes and find flaws in ancient reasoning. It would be a good thing to explain where the Greeks have made a mistake and why people still argue the case.

Zeno interpretation of the problem according to Wikipedia is:

*Simplicius has Zeno saying "**it is impossible to traverse an infinite number of things in a finite time**". This presents Zeno's problem not with finding the sum, but rather with f**inishing a task with an infinite number of steps**: how can one ever get from A to B, if an infinite number of (non-instantaneous) events can be identified that need to precede the arrival at B, and one cannot reach even the beginning of a "last event"?*[5][6][7][38]**.**

Well, in such case how it is possible the turtle moves say 10 meters while visiting infinitely many points? This is a necessary condition to talk about the race. There is no point going any further if one cannot accept the fact motion takes place. Whether or not we can logically deny impossibility of motion is the next thing.

There is no discrete steps in reality of the assumed continuous motion. There is an infinite number of waypoints from a larger set of infinite points which we just wish to see the objects passing through, and which are singled out by an arbitrary rule. The "steps" exist in our imagination. There is no infinite number of detectors in those points.

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Zeno's logic is flawed at its roots. If you reason about the motion using the argument: *"it is impossible to traverse an infinite number of points in a finite time" then* ***this*** **is not a valid argument until rigorously proven.** We can see some reasoning to argue the case.

There is a few idealizations necessary to prove it. One is simultaneous start, but the start does not need to be simultaneous for the problem to be still valid. Small delay does not change the problem. Non relativistic motion interpretation is also acceptable as this what everybody was using.

The proofs can be based on elementary kinematics but this would not be fair to Ancient Greeks to use it against them. The second kind is a common sense reasoning contradicting Zeno's conclusion. It could have been used in ancient Greece. I will start with that common sense argument.

1. Assume motion is possible and the reference points on Achilles and the turtle are in the middle of their bodies' horizontal cross-section while running are used for position scoring. You need those points to judge the completion of the race.

2. Everybody agrees that because Achilles is faster the distance gradually and continuously diminishes.

3. We notice that the turtle has its head well away in front of the middle of its body and the head moves with it at a fixed offset with respect to the middle.

4. While chasing the turtle, Achilles is also implicitly chasing the turtle's head without changing anything physical in the process; other than we are altering the mental focus on a different part of turtle's body. So for now we think of the head as the reference ignoring the middle for the time being.

5. Since the distance from Achilles to turtle reference points diminishes to smaller and smaller values, it does the same simultaneously with respect to the turtle's head. If the reference point of Achilles comes really close but not infinitely close to the turtle's head, Achilles's middle body has already been be past the turtle's body middle point (if the turtle is big enough e.g. from Galapagos).

6. That is proving that **in finite time we can reach points past turtle's middle point, using vague elementary concepts of motion - similar as that used by Zeno et. al.** All of this by just changing mental focus from the middle of the turtle's body to it's head.

7. Although the simple reasoning shows no substance in this specific Zeno's problem (which does not exist), it does not show exactly why arguments used by Zeno's proponents seem to indicate otherwise. In other words, where is an error in their reasoning? But this is a question to philosophers.

8. <u>The physical aspect of the chase does not depend what we are thinking about it and how we reckon waypoints. We comfortably go through infinite number of points.</u>

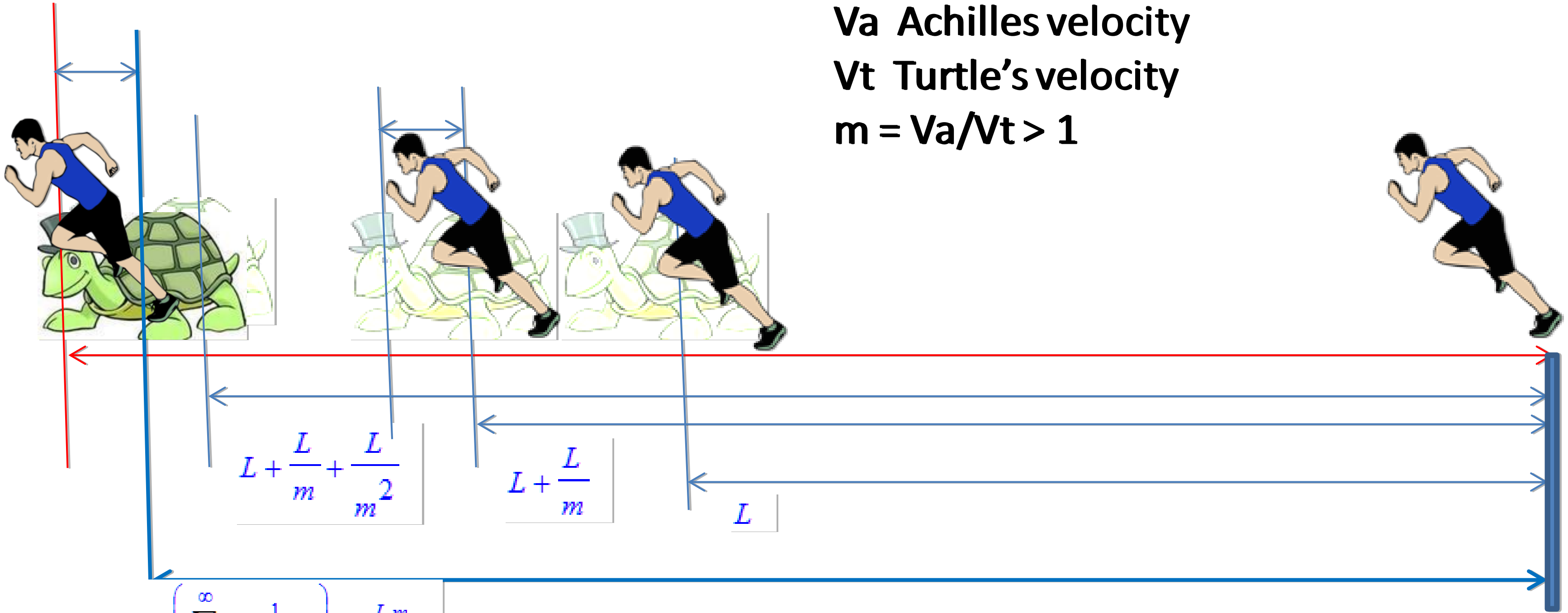
9. Below there is a numerical example. Data marked red indicate the moment the turtle is overtaken while focusing attention on its head. If we focus on the mid-point we will not have finite number of

waypoints, yet we would get there in time through infinite number of waypoints. The turtle’s head is 0.5 m in front of the middle of its body.

Time	Achilles	Head	Turtle	Distance to turtle's middle
0	0	100.5	100	100
10.05	100.5	150.75	150.25	49.75
15.075	150.75	175.875	175.375	24.625
17.5875	175.875	188.4375	187.9375	12.0625
18.84375	188.4375	194.7188	194.21875	5.78125
19.47188	194.7188	197.8594	197.359375	2.640625
19.78594	197.8594	199.4297	198.9296875	1.0703125
19.94297	199.4297	200.2148	199.7148438	0.28515625
20.02148	200.2148	200.6074	200.1074219	-0.107421875

Overtaking the turtle in finite time by approaching the middle of its head infinitely close

L Initial distance
Va Achilles velocity
Vt Turtle's velocity
m = Va/Vt > 1

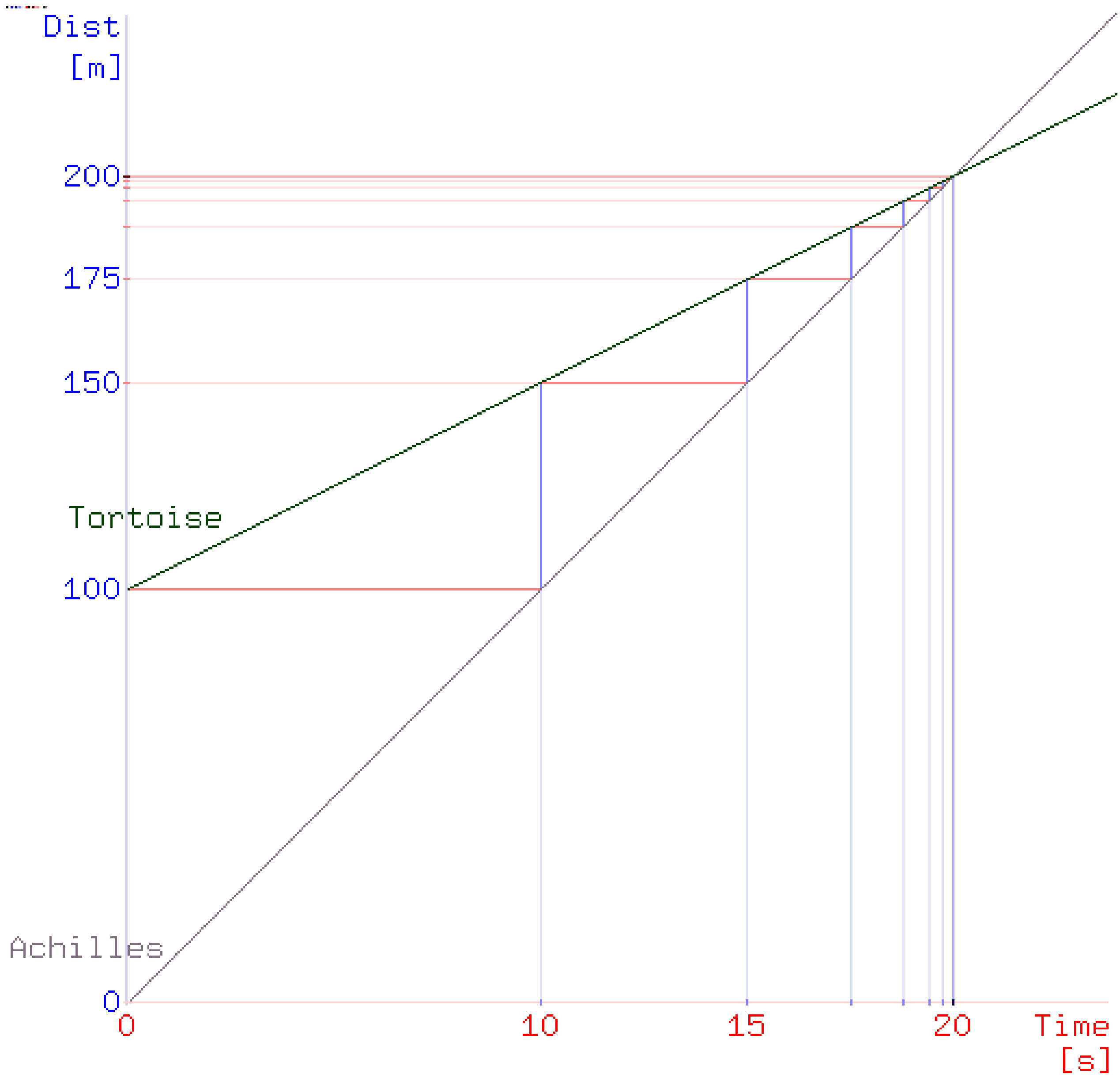


$$L\left(\sum_{n=1}^{\infty}\frac{1}{m^{(n-1)}}\right)=\frac{L\,m}{-1+m}$$

$$Time\_A := \frac{L\left(\sum_{n=1}^{\infty}\frac{1}{m^{(n-1)}}\right)}{Va}$$

The mathematical proof can provide a clear evidence that the infinite sum of time periods converges.

There are no steps in Zeno's scenario; there is a conceptual exposition of certain sequence of waypoints on the path of moving objects.
The illustration of the scenario is shown on the picture borrowed from Wikipedia:



The blueprint of a simplistic mathematical proof

The series of “steps” in this scenario via successive waypoints of interest is as follows:

From the initial separation **L,** Achilles runs to reach the position **L.** At this point, the next waypoint is the simultaneous position of the turtle while Achilles is at L, and so on and so on….

The table below shows a few steps in this scenario. It is clear that the infinite series of steps approaching the final position in this scheme yields infinites sums of a quite regular form.

The sums are constructed using the rule that positive velocity of Achilles **Va** is **m** times greater than that of the turtle. Subsequent position of Achilles at waypoints starting with **L** is the previous position of the turtle at the waypoint. The infinite sum convergence can be proven by deriving formulas for the partial sums. If the sequence of partial sums converges to a limit, then the series converge to the sum. This is elementary calculus from the secondary school. Unless one proves you cannot calculate limits there is no way to question the derived formulae

Time	Position *Achilles*	Position *Turtle*	*Distance*
0	0	L	L
$\frac{L}{Va}$	L	$L+\frac{L}{m}$	$\frac{L}{m}$
$\frac{L}{Va}+\frac{L}{m\,Va}$	$L+\frac{L}{m}$	$L+\frac{L}{m}+\frac{L}{m^2}$	$\frac{L}{m^2}$
$\frac{L}{Va}+\frac{L}{m\,Va}+\frac{L}{m^2\,Va}$	$L+\frac{L}{m}+\frac{L}{m^2}$	$L+\frac{L}{m}+\frac{L}{m^2}+\frac{L}{m^3}$	$\frac{L}{m^3}$
$\frac{L}{Va}+\frac{L}{m\,Va}+\frac{L}{m^2\,Va}+\frac{L}{m^3\,Va}$	$L+\frac{L}{m}+\frac{L}{m^2}+\frac{L}{m^3}$	$L+\frac{L}{m}+\frac{L}{m^2}+\frac{L}{m^3}+\frac{L}{m^4}$	$\frac{L}{m^4}$
…	…	…	..
The converging values for infinite number of waypoints which clearly converge to finite values in finite time			
$Time\_A := \frac{L\left(\sum_{n=1}^{\infty}\frac{1}{m^{(n-1)}}\right)}{Va} = \frac{L\,m}{Va\,(-1+m)}$	$L\left(\sum_{n=1}^{\infty}\frac{1}{m^{(n-1)}}\right)=\frac{L\,m}{-1+m}$	$L\left(\sum_{n=0}^{\infty}\frac{1}{m^{n}}\right)=\frac{L\,m}{-1+m}$	0

I do not see any merits in claims that Zeno Paradox is a problem today. If at all, it is the same problem as why anything exists, why things move, but this is an unknown not a paradox.