Beauty, Odds, and Credence

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1. Introduction

The following is one version of the so-called Sleeping Beauty problem.

Some researchers are going to put Sleeping Beauty to sleep on Sunday evening. She will sleep through most of the following week except on up to two occasions. The researchers will wake her up briefly either once—on Monday—or twice—on Monday and Wednesday—depending on the result of a fair coin toss that takes place after she goes to sleep on Sunday but before she wakes up on Monday: if it is heads she will be woken briefly on Monday and sleep through the rest of the week; if it is tails she will be woken up again on Wednesday. After each waking Sleeping Beauty will be put back to sleep with a drug that erases any memory of the waking. When Sleeping Beauty is woken up during the week, what should her level of confidence

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be that the outcome of the toss was heads? What should her level of confidence be upon learning that it is Monday?[1]

Let M, W, H, T be the propositions that it is Monday, that it is Wednesday, that the coin landed heads, and that the coin landed tails, respectively. There are three main positions in the literature. Thirders, led by Adam Elga (2000), hold that upon first waking up Sleeping Beauty's level of confidence that $M\&H$ ought to be equal to her level of confidence that $M\&T$; and her level of confidence that $M\&T$ ought to be equal to her level of confidence that $W\&T$. This means that upon waking up, Sleeping Beauty ought to be twice as confident that the coin landed tails as that it landed heads. When she finds out that it is Monday, her level of confidence that the coin landed heads ought to be updated to being equal to her level of confidence that the coin landed tails.

Halfers, led by David Lewis (2001), hold that upon waking up, Sleeping Beauty's level of confidence that H ought to be equal to her level of confidence that T. And her level of confidence that $M\&T$ ought to be equal to her level of confidence that $W\&T$. When she finds out that it is Monday, she ought to update her belief so that her level of confidence that H is twice her level of confidence that T.

Double Halfers (e.g., Bostrom 2007; Cozic 2011; Meacham 2008; Pust 2012), hold that Sleeping Beauty's levels of confidence when she wakes up ought to be as Halfers say they ought to be. But when she finds out that it is Monday, her updated levels of confidence ought to be as Thirders say. That is to say, her level of confi-

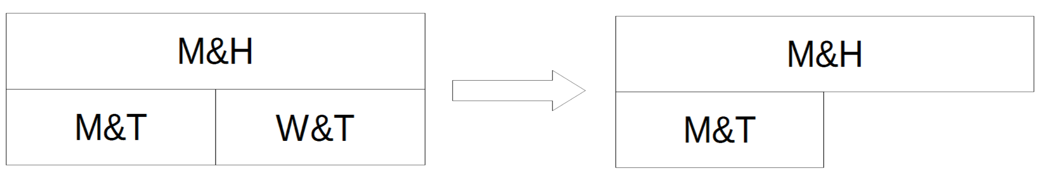
[1] It is often left open when the coin toss takes place as the timing of the toss is inessential to the problem. I stipulate it to be before Sleeping Beauty's first waking to make exposition easier but nothing hangs on this stipulation. I stipulate that the second waking occurs on Wednesday rather than Tuesday for the sake of less confusing abbreviations.

dence that the coin landed heads ought to be the same as her level of confidence that it landed tails throughout the week.

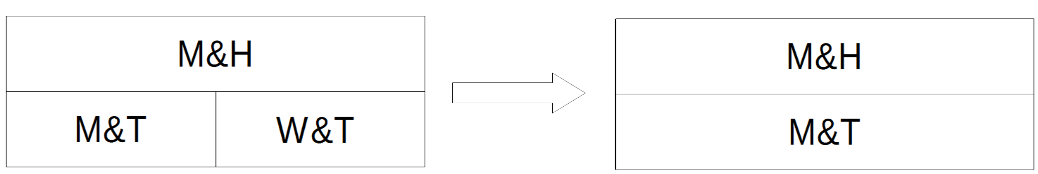
The following is a diagrammatic representation of the three positions. On the left hand side, the size of the blocks relative to each other represent Sleeping Beauty's levels of confidence relative to each other upon first waking up; and on the right hand side they represent her levels of confidence relative to each other upon learning that it is Monday.

Thirder

Halfer



Double Halfer



Which, if any, of the three positions is the correct position? In the following I will be considering some bets that Sleeping Beauty might be offered. Suppose that upon waking she is offered a bet: her stake is a dollar and she will make a certain amount of profit if and only if the coin landed heads. What is the fair amount of profit for this bet—that is, what are the fair betting odds? I will argue that Double Halfers can give the most satisfactory answer to this and related questions.

Some readers might be suspicious of approaching epistemological problems through pecuniary considerations that guide betting odds. I share some misgivings of this sort and I do not wish to suggest that epistemological questions just are decision theoretic problems typified by discussions of fair betting odds. However, what an agent ought to regard as rationally acceptable betting odds is a function of what risks and opportunities she rationally takes herself to be facing. In the kinds of bets under consideration in this paper, the risks and opportunities as perceived by Sleeping Beauty are a function of the levels of confidence Sleeping Beauty ought to have about the outcome of the coin toss. If it can be shown that this or that position about the Sleeping Beauty problem gives rise to objectionable assessments of risks and opportunities, that is a reason—though not necessarily a decisive one—to question the position. Pointing out that Thirders and Halfers face this sort of problem, whereas Double Halfers do not, is my main concern in this paper.

2. Fair Odds

A few remarks on notation before proceeding. I will use $C(P)$ to represent the level of confidence that P. I will be using British odds for representing betting odds.

And when I speak of 'odds', I mean *betting* odds. British odds represent the ratio between potential net profit from the bet and the stake. When a bet has odds $m : n$, m represents the net profit from the bet's payout in case of winning the bet when the stake is n. For instance, if the odds are $4 : 3$ and you pay 3 dollars for the bet, you will receive a payout of 7 dollars for a net profit of 4 dollars (7 dollars payout minus the initial outlay of 3 dollars). Since the profit for each unit of stake is given by m/n, I generally will give odds in the form $k : 1$.

When the odds for a bet are fair, a rational agent ought to be just as willing to buy the bet as to sell the bet. For instance, given fair odds $k : 1$ for a bet that P is the case, she ought to be just as willing to pay a dollar now for the right to receive a profit of k dollars in case that P, as to receive a dollar now in return for the obligation to pay out a sum resulting in a net loss of k dollars in case that P.[2]

Here is an intuitively plausible way of thinking about rationally acceptable odds. Suppose you are considering whether or not to engage in a course of action. The course of action will result in some loss if P is false. Should you engage in the course of action? That depends on what happens if P is *true*. If that is not associated with any gains and refraining from the course of action does not have its own downsides, then you should not engage in it. On the other hand, if the course of action results in some gains in case P is true, then two things matter: what is the ratio between the potential gain to potential loss; and what is your level of confidence that P is true? The less confident you are that P is true, the more you need in potential gains relative to your potential losses. Concretely, the minimum ratio between potential

[2]Since you gain 1 dollar in case $\neg P$ and lose k dollars if P, this is, as far as the bottom line is concerned, the same as buying a bet that you win in case $\neg P$ at odds $1 : k$, or $1/k : 1$.

gains and losses is given by:

$$\text{net gain in case P : net loss in case } \neg P = C(\neg P) : C(P)$$

For instance, if you are as confident that P is true as that it is false, your potential gains must equal your potential losses. If you are half as confident that P is true as that it is false, your potential gains must be twice your potential losses. This, of course, is standard fare. Let us call this the *Basic Principle*.

In simple cases, the net loss in case $\neg P$ given a bet on P just is the price you pay the bookie—i.e., the stake—and the net gain in case P is the total payout of the bet minus the stake. In such a case, the fair odds are given by $C(\neg P) : C(P)$. But there can be situations in which the net loss in case $\neg P$ is different from the stake for the bet. For instance, the government—as distinct from the bookie—might encourage placing bets by handing out money to punters who lose (but not to winners). Since this is a government handout, it does not affect the bookie's expenses. The handout will reduce the net loss in case $\neg P$ and the Basic Principle tells us that the punter can go for odds that are lower than $C(\neg P) : C(P)$. Concretely, if the government were to cover half a punter's losses, a bet on a fair coin toss landing heads that costs a dollar will result in a net loss of half a dollar if the coin lands tails. That means that the net gain in case heads need only be half a dollar. The punter should still be willing to pay the bookie a dollar for the bet so that the odds are $1/2 : 1$ even though he is as confident the coin will land heads as that it will land tails: the punter can pay the bookie one dollar because he knows that if he loses the government will take out some of the sting by giving him half a dollar. And, of course, he would be happy to sell the bet at these odds. So with the government handouts to losers, odds $1/2 : 1$ are fair for a bet on the result of a fair coin toss! Of course, similar manipulations can take place on the payout side.[3] So we must be careful not to identify the ratio specified by the Basic Principle with the fair odds for a bet on P. This is a point that will be of some importance later.

3. Betting Odds Considerations for Sleeping Beauty

Let us now move to the Sleeping Beauty problem. In this section, I will consider two bets that a bookie might offer Sleeping Beauty and the fair odds for those bets. I will then argue in the following sections that Thirders and Halfers have difficulties giving good accounts of what the fair odds for those bets are, whereas the Double Halfers do not.

3.1. The Early Bet

Suppose Sleeping Beauty is offered a bet every time she wakes up. Thus, it is offered on Monday morning and if the coin landed tails the bet is also offered on Wednesday. The terms are: she pays a dollar now for a bet that results in profit of k dollars if and only if the coin toss before her waking landed heads. Call this the *Early Bet* (there is going to be a *Late Bet*, too). Further suppose that Sleeping Beauty knows that the bet is offered every time she wakes up. What must k be for the odds to be fair?

For simplicity's sake, let us assume that Sleeping Beauty will reject the bet if and only if the odds are unfair. And let us also assume that the bookie will keep on offering alternative odds until she accepts the bet. So if there are any fair odds for

[3] Notice that bookies have powerful reasons to lobby for such subsidies for punters: not only will the subsidies tend to encourage betting, they encourage placing bets at odds that are lucrative for bookies. There indeed are tax and other regulations that have this effect—typically more subtle than simple handouts—but that is beyond the scope of this paper.

the Early Bet, she will accept the bet at some odds on both Monday and Wednesday.

Everyone should hold that the fair odds for the Early Bet are $2 : 1$; i.e., $k = 2$. There are compelling arguments in favor of these odds that are independent of Sleeping Beauty's levels of confidence upon waking up on Monday that the coin landed heads.

The first argument is frequentist. Let us consider which odds Sleeping Beauty ought to accept if the experiment were repeated a large number of times. There are indefinitely many possible long term financial results. However, since Sleeping Beauty knows the coin is fair, if N is the number of weeks, she is certain that as N gets larger the long term result is more and more likely to be close to $\frac{1}{2} \times N \times k - \frac{1}{2} \times N \times 2$—during half of the weeks she gains k dollars, during the other half she loses two dollars by placing two losing bets: once on Monday and again on Wednesday. This entails that if k is anything other than 2, Sleeping Beauty either will be nearly certain that she will make a profit in the long term, or she will be nearly certain that she will make a loss in the long term, where the expected profits or losses increase with the number of times the experiment gets repeated. Such arrangements cannot be fair: it is either unacceptable to the bookie or unacceptable to Sleeping Beauty. For the bet to be fair, the expected long term results must be to break even, and that requires $k = 2$. Thus, the fair odds for the Early Bet are $2 : 1$.

Perhaps you are too much of a Bayesian to be moved by such apostate frequentist considerations. Here is another argument in favor of $2 : 1$. It is a diachronic Dutch Book argument originally due to Hitchcock (2004).

Suppose the stakes for the Early Bet placed on Monday and on Wednesday (if there is a bet) are 1 dollar each time and that the odds are $k : 1$. If the coin landed tails, Sleeping Beauty loses a total of 2 dollars over the week. If the coin landed heads, she gains k dollars. Suppose the bookie offers her an additional bet on Sunday, before she goes to sleep. She wins the bet if the coin lands tails, and loses if it lands heads. Since she knows the coin is fair, she ought to accept odds $1 : 1$ for the bet on Sunday. Let the stakes for this bet be s dollars. If the coin lands heads, Sleeping Beauty loses s dollars from the bet on Sunday and gains k dollars from the Early bet for a net result of $k - s$ dollars. This is negative just in case $s > k$. On the other hand, if the coin lands tails, she gains s dollars from the bet on Sunday and loses 2 dollars from the two Early Bets placed on Monday and on Wednesday for a net result of $s - 2$ dollars. This is negative just in case $s < 2$. Thus, the bookie can guarantee a loss for Sleeping Beauty by choosing the stake s for the bet on Sunday such that $k < s < 2$. This is possible if and only if $k < 2$. If $k > 2$, the bookie can guarantee a loss for Sleeping Beauty by reversing the direction of each bet and choosing a stake for the Sunday bet such that $2 < s < k$. Notice that all the bookie needs to know to guarantee Sleeping Beauty's loss is what odds she will regard as fair for the Early Bet. He and Sleeping Beauty can both know that. So there is no exploitation of information asymmetry here. In order to avoid susceptibility to a diachronic Dutch Book, k must be equal to 2. Thus, the fair odds for the Early Bet are $2 : 1$.[4]

Neither the frequentist nor the diachronic Dutch Book argument makes any assumptions about what Sleeping Beauty's levels of confidence ought to be upon waking up. The frequentist argument merely requires that the coin be in fact fair. The diachronic Dutch Book argument only requires that the fair odds for the bet on Sunday be $1 : 1$. So whatever your position on the original Sleeping Beauty problem, you ought to accept $2 : 1$ as fair for the Early Bet. Let me call the two arguments

[4]Briggs (2010) argues for the same point though in a somewhat different manner.

given in this section the *independent* arguments in favor of odds $2 : 1$ as fair for the Early Bet.

3.2. The Late Bet

Consider now a different bet. Suppose there is no Early Bet but that upon learning it is Monday Sleeping Beauty is offered a bet that the coin landed heads. Call this the *Late Bet*. What are the fair odds for this bet? Again, let us assume for simplicity's sake that Sleeping Beauty will reject the bet if and only if the odds are unfair, and that the bookie will keep on offering alternative odds until she accepts the bet.

Everyone should accept that the fair odds for the Late Bet are $1 : 1$. Let me again give two arguments.

The first argument is frequentist. Suppose the experiment is repeated multiple times. Each week on Monday, after an initial period of ignorance, Sleeping Beauty is told that it is Monday and then offered a bet that the coin landed heads. Since she is always woken up on Mondays, the number of bets she will place is the same as the number of coin tosses. Since the coin is fair, she can expect to lose half of the bets, and to win half of the bets. In order to be confident of breaking even in the long term, the odds need to be $1 : 1$ each time she places the Late Bet. Thus, the fair odds are $1 : 1$ for the Late Bet.

The second argument is a diachronic Dutch Book argument due to Draper and Pust (2008). Let the odds for the Late Bet be $k : 1$. Since the bet is offered only on Monday, she loses one dollar over the week in case the coin landed tails, and gains k dollars in case it landed heads. Suppose the bookie offers her an additional bet on Sunday, before she goes to sleep. She wins if the coin lands tails, and loses if it lands heads. Since she knows the coin is fair, she ought to accept odds $1 : 1$ for the bet on Sunday. Let the stakes for this bet be s dollars. Her net financial results from the two bets will be $k - s$ dollars in case the coin landed heads, and $s - 1$ in case the coin landed tails. If Sleeping Beauty accepts $k < 1$ for the Late Bet, the Bookie can guarantee a loss by choosing s such that $k < s < 1$. If she accepts $k > 1$, the bookie can reverse the direction of the two bets and make sure that $1 < s < k$ to guarantee a loss for Sleeping Beauty. The only odds that avoid this are $1 : 1$. Thus, the fair odds for the Late Bet are $1 : 1$.

Notice that, just as the independent arguments about the Early Bet's fair odds, neither the frequentist nor the diachronic Dutch Book argument about the Late Bet makes assumptions about what Sleeping Beauty's levels of confidence ought to be upon learning that it is Monday. So whatever your position on the Sleeping Beauty Problem, you ought to accept $1 : 1$ as fair for the Late Bet. Let me call the two arguments given in this section the *independent* arguments in favor of odds $1 : 1$ as fair for the Late Bet.

4. Problems for Thirders

There is a problem for Thirders. They have difficulties maintaining that $2 : 1$ are fair for the Early Bet. To see this, consider the following. If the coin toss landed tails, Sleeping Beauty will be offered the Early Bet twice, on Monday and on Wednesday. That is to say, if she loses the bet once, she will lose twice. The odds she accepts on Wednesday make no difference to the total amount she loses over the week. Suppose that on Monday she accepts a bet that the coin landed heads at odds $2 : 1$. She pays a dollar for the bet. If the coin did land heads, she earns 2 dollars over the course of the week. If the coin landed tails, she will lose 1 dollar on Monday, and 1 dollar again on Wednesday for a total loss of 2 dollars. This is something Sleeping Beauty can figure out for herself. Now, a Thirder holds that upon waking up Sleeping Beauty ought to be twice as confident that the coin landed tails as that it landed heads. But that means that if she accepts odds $2:1$ she ought to be twice as confident that she will lose 2 dollars over the course of the week as that she will gain 2 dollars over the course of the week. That does not look like an acceptable deal. The Basic Principle tell us that she ought to demand that the gains from winning the bet on Monday be twice her potential net losses; i.e., the net gains must be 4 dollars. So Thirders should hold that the fair betting odds for the Early Bet are $4:1$.

More generally, we can look at what will happen over the course of the week as follows. Given odds $k:1$ for the Early Bet, if the stakes are 1 dollar each time the bet is placed, the gains over the course of the week in case the coin landed heads are k dollars, whereas the losses over the week in case the coin landed tails are 2 dollars. The Basic Principle tells us:

$$k:2 = C(T):C(H)$$

Thus,

$$k = \frac{2 \times C(T)}{C(H)}$$

As already noted, this yields $k = 4$ for Thirders. This approach to determining fair odds looks at the entire week and the value of the set of bets—a book—placed over the week. Of course, how many bets are in the set depends on the result of the coin toss but the crucial point here is that it is not looking at one bet in isolation. Let us call this the *Bookie Approach*.

The problem for Thirders is that the result of the Bookie Approach is in blatant conflict with the results of the independent arguments for odds $2:1$ as fair. What gives?

There is a principled way of rejecting the Bookie Approach. That is to adopt what is known as causal decision theory. Hitchcock (2004) and Draper and Pust (2008) are examples of Thirders taking this approach to the Early Bet. According to causal decision theory, one ought to assess outcomes in terms of the causal consequences of the agent's decision. The crucial point is that there can be consequences that one can foresee that are not causal consequences of the agent's decision. In the case of the Early Bet, for instance, Sleeping Beauty has evidence that if she loses the bet on Monday, she will lose the bet on Wednesday, but that does not mean that her placing the bet on Monday *causes* her placing another bet on Wednesday. Equipped with this thought, we can reason as follows.

When Sleeping Beauty accepts the Early Bet at odds $k : 1$, there are three possible causal consequences. Either it is Monday and she wins k dollars; or it is Monday and she loses 1 dollar; or it is Wednesday and she loses 1 dollar. The last two possibilities are both cases in which she loses her stake of 1 dollar, and her level of confidence that either one or the other of them is the or case is simply her level of confidence that the coin landed tails. The Basic Principle tells us that the fair odds for the Early Bet are given by:

$$k : 1 = C(T) : C(H)$$

and thus,

$$k = \frac{C(T)}{C(H)}$$

Let us call this the *Causal Approach* to the fair odds for the Early Bet. For Thirders, this approach yields $2 : 1$ as fair for the Early Bet.

Are Thirders out of the woods? Maybe all they have to do is to insist that causal decision theory is the correct approach to decision making? No.

Notice that by embracing causal decision theory, Thirders lose the ability to appeal to the independent arguments for $2:1$ to support the result of the Causal Approach. In the frequentist argument one appeals to the aggregate result of multiple decisions, but a decision during any one week does not *cause* decisions and their consequences during the other weeks. In the diachronic Dutch Book argument one appeals to the collective result of up to three bets—Sunday, Monday, and Wednesday—and, again, a decision on one of the days does not cause the decisions and results on the other days; in particular, the decision to place a bet on Monday does not cause the loss on Wednesday. That is why adopting causal decision theory yields the formula for the Causal Approach above. So the predictable financial results that these arguments appeal to cannot play a role in rational decision making if causal decision theory is taken seriously because they must be seen as irrelevant to decision making. And insisting that predictable consequences are irrelevant unless they are causal consequences of the choice at hand is crucial to causal decision theorists for one well-known feature of causal decision theory is that it recommends the financially worse option in Newcomb's Problem (Horwich 1987). What saves causal decision theory, according to its defenders, is that the worse financial outcome in Newcomb's Problem is not a causal consequence of the decision thus making that outcome irrelevant to the decision at hand. If that is the attitude one takes towards non-causal financial consequences in one case, one must also take the same attitude in the Sleeping Beauty case. So Thirders who appeal to the Causal Approach to maintain $2:1$ are fair for the Early Bet lose the support of the independent arguments in favor of these odds.

Thus, Thirders who are causal decision theorists can avoid getting Dutch Booked in the Sleeping Beauty case as Briggs (2010) points out. However, that point cannot be appealed to by the same Thirders in defense of their position: it is a defense that is available only to those who are *not* causal decision theorists. Notice how peculiar the situation is. I, who do *not* believe in causal decision theory, am impressed by the independent arguments in favor of $2 : 1$, and I can recommend that Thirders embrace causal decision theory on the grounds that doing so will lead to good financial results for the Early Bet. But I must also hold that Thirders who embrace causal decision theory are making a mistake. A happy mistake, to be sure, since the mistake leads to the avoidance of being Dutch Booked, but I who am impressed by the independent arguments must reject causal decision theory as the correct approach to decision making even if at times it delivers the right results.

This means that Thirders who accept the independent arguments in favor of $2 : 1$ and also embrace causal decision theory have an inconsistent position: in accepting the independent arguments, they embrace a theory they must reject.

Of course, Thirders might embrace causal decision theory not because of the independent arguments, but because of some other considerations. But that weakens their dialectical position. Halfers who embrace causal decision theory have to say that the fair odds for the Early Bet are $1 : 1$ as can be seen by plugging in the numbers in the formula above for the Causal Approach. But causal decision theorists cannot use that as a complaint against Halfers since they cannot appeal to the independent arguments. That means that a causal decision theorist cannot appeal to the very real financial risks that arise from placing bets at certain odds as a consideration for or against accepting those odds. That is not a happy position to be in.

5. Problems for Halfers

Let us take a look at Halfers. Halfers hold that upon waking up Sleeping Beauty ought to be as confident that the coin landed heads as that it landed tails. And when she learns it is Monday, she ought to be twice as confident that the coin landed heads as that it landed tails. So for Halfers, the Bookie Approach will yield $2:1$ as fair for the Early Bet as can be seen from the formula given above for the Bookie Approach:

$$k = \frac{2 \times C(T)}{C(H)}$$

This is as it should be: according to Halfers, given odds $k:1$ she is as confident of losing two dollars as of gaining k dollars over the week.

As noted earlier, the Causal Approach to the Early Bet will yield $1:1$ for Halfers but Halfers have a straightforward explanation why the Bookie Approach is to be favored: In the Early Bet, the situation is such that if Sleeping Beauty loses the Early Bet once, she will lose it twice, but she will win only once. The Causal Approach, but not the Bookie Approach, fails to take this into account because the first losing bet does not cause the second losing bet. This results in the Causal Approach undercounting the net losses in case the coin landed tails.

You will notice that the odds Halfers should accept on the Bookie Approach is *not* $1:1$ even though she is as confident that the coin landed heads as that it landed tails. Is that not in conflict with the Basic Principle? It is not. A pointed out earlier, the Basic Principle governs the ratio between the net loss in case $\neg P$ and the net gain in case P. Betting odds, on the other hand, specify the ratio between the stake for a particular bet on P and the potential net payout of that same bet. The two can come apart because in some situations the net losses in case $\neg P$ are different from the stake for a bet on P and this can happen without affecting your levels of confidence in P and $\neg P$. In the example I gave earlier, the net loss was smaller than the stake because of a government handout to losers. But one can easily think of ways of making the net losses larger: e.g., a tax levied on punters who lose (but not on winners).

In Sleeping Beauty's case, she knows that the net loss she faces if the coin landed tails is 2 dollars. Since she is as confident that the coin landed heads as that it landed tails, the Basic Principle tells us that the net gain if the coin landed heads must be equal to her net losses, i.e., the net gain must be 2 dollars. Since she can only make a gain through winning the Early Bet once, the net payout for that winning bet must be 2 dollars. Its stake is 1 dollar—the net loss of two dollars in case the coin landed tails arises from placing this bet and another losing bet of the same price. Thus, the odds she accepts for the bet on offer must be $2:1$ so that she gains 2 dollars *if* she wins the bet, even though she is as confident that the coin landed heads as that it landed tails.

Bradley and Leitgeb (2006) note this point but their exposition makes it seem as though some peculiar epistemic situation is needed to bring about a discrepancy between the betting odds and levels of confidence. That is not so, as the government-handout-for-losers example above make clear. What matters is that sometimes the agent can know that the losses she faces in case $\neg P$ are different from the stake for the bet on P, and in such a case the Basic Principle demands that the *betting odds* diverge from the ratio between the levels of confidence that $\neg P$ and that P.

Thus, as far as the Early Bet is concerned Halfers do not have any difficulties. But matters are different when it comes to the Late Bet. The Late Bet is a bet offered to Sleeping Beauty upon her learning that it is Monday. She wins k dollars if the coin landed heads, and loses her stake otherwise. According to Halfers, Sleeping Beauty ought to be twice as confident of winning the Late Bet as of losing it. But this means, given the Basic Principle, that Halfers need to hold that the fair odds for the Late Bet are $1/2 : 1$. And this is in conflict with the result of the independent arguments that everyone should accept that the fair odds for the Late Bet are $1 : 1$.

Halfers do not seem to have much room to maneuver here. Insisting that $1/2 : 1$ are in fact fair would require denying the cogency of the independent arguments. Halfers could do that by insisting on causal decision theory. But that would require accepting that $1 : 1$ are the fair odds for the Early Bet which does not help the Halfers' case.

Do Halfers have grounds for rejecting $1/2 : 1$? That looks difficult as pointed out by Draper and Pust (2008). As noted above, Halfers can explain why the Bookie Approach is to be favored over the Causal Approach in the case of the Early Bet. But no such explanation is available in the case of the Late Bet. The bet is placed exactly once during the week—on Monday—and Halfers hold that Sleeping Beauty ought to be twice as confident of winning it as of losing it. What goes wrong in concluding that therefore the fair odds are $1/2 : 1$? Without an answer to this, Halfers would only have an ad hoc rejection of an otherwise perfectly acceptable way of determining fair odds.

Thirders do not have a problem with the Late Bet. Since, according to them, upon learning it is Monday Sleeping Beauty ought to be as confident that the coin landed heads as that it landed tails, she ought to accept $1 : 1$ as fair—she is as confident of winning the Late Bet as of losing it. But, as argued above, Thirders have difficulties with the Early Bet.

The upshot so far is that Halfers have no difficulties with the Early Bet but have difficulties with the Late Bet. Thirders, on the other hand, have difficulties with the Early Bet but no difficulties with the Late Bet.

6. Another Problem for Both: Maximin Approach

I want to raise one more concern for both Halfers and Thirders. Consider again the Early Bet and how Sleeping Beauty ought to go about choosing the odds. Here is an approach that I will call the *Maximin Approach*. Sleeping Beauty knows that if it is in fact Wednesday, her choice of odds makes no difference to the overall outcome: she will lose the bet on Wednesday and therefore add to her losses. The only chance she has of improving her prospects are on Monday. So she ought to choose odds that are the right ones in case it is in fact Monday. Let her assume that it is Monday and figure out what odds to accept. Notice that this is not to assume that the bet is the Late Bet discussed earlier; the case still involves her memory being erased later and her being offered the same bet on Wednesday in case the coin landed tails. On the assumption that it is Monday, she knows she will lose two dollars in total if she loses the bet on Monday (a dollar on Monday and another on Wednesday); and she knows she will gain a total of k dollars if she wins the bet on Monday. What are the levels of confidence that Sleeping Beauty should use to decide what odds to accept on the assumption that it is Monday? The natural response is that she ought to proceed as if she knows that it is Monday. Let $U_M(T)$ and $U_M(H)$ be the updated rational levels of confidence in T and H respectively given knowledge that M. The Basic Principle tells us:

$$k : 2 = U_M(T) : U_M(H)$$

and thus,

$$k = \frac{2 \times U_M(T)}{U_M(H)}$$

This yields $k = 2$ for Thirders, and $k = 1$ for Halfers. So the fair odds for the Early Bet according to the Maximin Approach are $2 : 1$ for Thirders, and $1 : 1$ for Halfers.

Notice that the Maximin and Bookie Approaches deliver different verdicts for Thirders and Halfers—the Bookie Approach delivers $4 : 1$ for Thirders and $2 : 1$ for Halfers. In general, the kind of reasoning typified by the Maximin Approach does not deliver the same result as the kind of reasoning typified by the Bookie Approach. However, in this particular case, the two should deliver the same result. Let me explain. On the Bookie Approach, one looks at the expected gains and losses over the whole week and chooses potential net gains for the bet on Monday so that Sleeping Beauty does not have to expect to be down over the whole week. This just is to choose the odds for the bet on Monday so that the potential gains from winning on Monday can offset the potential losses from losing both on Monday and on Wednesday. And this is precisely what the Maximin Approach seeks to do as well. On the face of it, then, the Bookie and Maximin Approaches ought to deliver the same result. Thirders and Halfers must therefore explain how it is that the Bookie and Maximin Approaches come apart in this case. Moreover, since $2 : 1$ do seem to be the actual fair odds for the Early Bet, Thirders need to argue that the Maximin Approach gets it right and not the Bookie Approach; Halfers, on the other hand, need to argue the opposite. It is not clear to me what either argument would look like. Most likely, it would have to be an argument to the effect that the notion of *offsetting* potential losses is ambiguous so that offsetting potential losses in the

Bookie Approach is something different from offsetting potential losses in the Maximin Approach. But the two senses of *offsetting* will have to be produced first and then Thirders must give an argument in favor of one over the other as relevant for choosing the odds, and Halfers have to argue that Thirders have it backwards. The prospects for any such argument seem dim. As I pointed out, for this particular case at hand the Bookie and Maximin Approaches seem to be two ways of describing the same strategy. One must suspect that Thirders and Halfers do not have coherent positions.

7. Double Halfers to the Rescue

Let me now turn to the Double Halfer position. Discussing the virtues of the Double Halfers position will be brief. It is just that they do not face the difficulties the others face.

Let me restate the formulae for the fair odds of the Early Bet that different approaches yield.

The Causal Approach:

$$k = \frac{C(T)}{C(H)}$$

The Bookie Approach:

$$k = \frac{2 \times C(T)}{C(H)}$$

The Maximin Approach:

$$k = \frac{2 \times U_M(T)}{U_M(H)}$$

Double Halfers hold that $C(T) = C(H)$ and that $U_M(T) = U_M(H)$. Thus, for them the Causal Approach results in $1 : 1$ as fair for the Early Bet, the Bookie Approach results in $2 : 1$, and the Maximin Approach in $2 : 1$ as well.

Double Halfers can reject the result of the Causal Approach on the same grounds as Halfers: the Causal Approach fails to take into account that if Sleeping Beauty loses the Early Bet once, she will lose it twice. The Bookie Approach and Maximin Approach both result in $2 : 1$ so that there is no need to worry about which Approach is to be preferred, and, crucially, the result is in line with the independent arguments for $2 : 1$ as fair for the Early Bet.

What about the Late Bet? Since Double Halfers hold that, upon learning that it is Monday, Sleeping Beauty ought to be as confident that the coin landed heads as that it landed tails, Double Halfers can hold unproblematically that the fair odds for the Late Bet must be $1 : 1$.

Double Halfers get it right for the Early Bet on both the Bookie, and Maximin Approaches. Double Halfers also get it right for the Late Bet. This means they do not face the explanatory burdens and worries about coherence that Thirders and Halfers face. Surely, that is a reason to favor Double Halfers over Thirders and Halfers.

Before concluding, let me remark a little on the nature of the arguments against Halfers and Thirders. My arguments are not Dutch Book arguments because they do not show that either position must recommend odds that are susceptible to Dutch Books. Both of them could appeal to the independent arguments to arrive at the correct odds for both the Early and Late Bets. After all, those arguments are independent of the particular position one takes on Sleeping Beauty's rational levels of confidence during the week of the experiment. The trouble for the two positions arises when one considers alternative, seemingly fine, ways of arriving at the fair odds. In particular, trouble arises when one thinks in terms of expected utility. Thirders need to take a particular approach known as causal decision theory in order to arrive at the correct odds for the Early Bet. In doing so, they must reject the independent arguments as fallacious and thereby lose an important ground for holding the correct odds really are correct, or else live with a contradiction in their overall view of the Sleeping Beauty problem. Halfers can arrive at the correct odds for the Early Bet by rejecting causal decision theory, but that does not enable them to arrive at the correct odds for the Late Bet via considerations of expected utilities. So Halfers will be stuck with arguments that deliver conflicting verdicts as to the fair odds for the Late Bet. They could, of course, embrace causal decision theory and insist that the independent arguments get it wrong, but that is hardly a convincing position. The points regarding the Maximin Approach compound the difficulties with overall coherence for both Thirders and Halfers. The fundamental problem that my arguments show is that of ensuring that various seemingly fine approaches to the fair betting odds deliver the same answers, and to explain satisfactorily which approaches must be rejected in cases they do not all arrive at the same fair odds. As pointed out, Double Halfers have the easiest time as far as handling the betting odds in the Sleeping Beauty case is concerned.

8. Conclusion

I have argued that neither Thirders nor Halfers can give satisfactory accounts of what the fair betting odds are for the Early Bet and the Late Bet. Only Double Halfers avoid the problems that Thirders and Halfers face. And that is a reason to favor Double Halfers over Thirders and Halfers. Of course, this need not be decisive. There are objections to Double Halfers that I have not addressed at all. In particular, Double Halfers must recommend a deviation from conditionalization when it comes to Sleeping Beauty's updating her levels of confidence upon learning it is Monday and this requires further defense. Perhaps, the Double Halfer position is not the best position everything considered. But the betting odds considerations presented here in favor of Double Halfers, and against Thirders and Halfers, must be part of everything to be considered.

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