It Can Be Rational to Change Priors

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Abstract

According to a widely held norm of rationality, one should not change prior credences without new evidence. An important argument for this norm appeals to accuracy considerations, which says that changing priors doesn't maximize expected accuracy. This is because accuracy measures are strictly proper, and thus any probabilistically coherent person regards her own priors as uniquely maximizing expected accuracy compared with other priors.

This paper attempts to resist the accuracy argument against changing priors. We argue that even if rational epistemic decisions maximize expected accuracy according to strictly proper accuracy measures, it can still be rational to change priors sometimes. The core idea of our argument is that changing priors can be rational if one wants to maximize not just one's current, short-term accuracy but also future, long-term accuracy. Our argument, if successful, shows that considering long-term accuracy has significant ramifications for the accuracy-first project.

Keywords: Bayesian Epistemology; Epistemic Utility; Prior Probabilities

1 Introduction

According to a widely held norm of rationality, one should not change beliefs without new evidence. In the Bayesian framework that models beliefs as obtained by conditioning a prior credence function on evidence, the norm says that one should not change priors. This norm has been motivated in a variety of ways.

Before listing the motivating arguments, we should first clarify the notion of prior. The notion has been used in at least two different ways in the literature. Sometimes, it means the credence held at the time before receiving a given piece of evidence. Priors in this sense are fixed and cannot intelligibly change. Other times, it means some hypothetical credence function that serves as one's epistemic standards, and one's actual credence at a time is supposed to be determined by conditioning this hypothetical credence on one's new evidence at the time. Priors in this sense can intelligibly change. In this paper, we use the notion of prior in the second way.[1]

Here are some existing arguments against changing priors. First, changing priors violates conditionalization: if one gains no new evidence, one should conditionalize on a tautology, which will result in the same priors (Meacham, 2014, 2015). Second, changing priors violates conservatism, which forbids changing beliefs without specific reasons to doubt them (Harman, 1995; Vahid, 2004, p. 97). Third, there is a practical argument, which says that having stable priors is critical for carrying out one's long-term plans (Titelbaum, 2015). Finally, and most importantly, there is an argument based on accuracy considerations, which says that changing priors doesn't maximize expected accuracy. This is because accuracy measures are strictly proper, and thus any probabilistically coherent person regards her own priors as uniquely maximizing expected accuracy compared with other

[1]Thanks to an anonymous reviewer for the urge to clarify the notion of prior.

priors (Schoenfield, 2014, forthcoming).[2]

It's not too hard to find ways to resist some of the above arguments. For instance, the conditionalization argument only works for some formulations of conditionalization but not for others.[3] The argument from conservatism is debatable if conservatism is questionable (Christensen, 1994). And the practical argument can be doubted as to whether it shows that changing priors is irrational in the epistemic sense of rationality; it also doesn't show that changing priors is always irrational, because sometimes you have no plans hanging on the belief in question.

The accuracy argument, on the other hand, may strike many as harder to resist. The argument rests on three assumptions: accuracy is the primary goal in epistemic decisions, accuracy measures should be strictly proper, and rational decisions should maximize expected utility. All three assumptions, while not uncontroversial, are broadly accepted in the present epistemic utility theory literature.

This paper tries to resist the accuracy argument against changing priors without challenging the three above-mentioned assumptions. We argue that even if accuracy is the primary goal in epistemic decisions, and even if rational epistemic decisions should maximize expected accuracy according to strictly proper accuracy measures, it can still be rational to change priors some times. The core idea of our argument is that changing priors can be rational if one wants to maximize not just one's current, short-term accuracy but also future, long-term accuracy. We explain this argument in Section 2, answer an objection in Section 3, and conclude in Section 4 that considering long-term accuracy has significant ramifications for the accuracy-first project.

[2]For additional arguments against changing priors, see Titelbaum (2015).

[3]More specifically, it doesn't work for the wide-scope formulation of conditionalization (Meacham, 2016).

2 Changing Priors to Be More Accurate in the Future

Suppose that you are ordering lunch at a restaurant. You have two options: a chicken salad or a duck-egg soup with tofu, and all you care about is the taste. You've had the chicken salad before, and you know that it tastes good. You've never had the duck-egg soup before, but the menu photos tell you that you will either love it or hate it. For concreteness, let's suppose that you are certain that the utility of the chicken salad (where the utility is entirely based on gustatory pleasure) is 1; the utility of the duck-egg soup is either 5 or -5, and your confidence in the two possibilities is even. Let's also assume you'll return to this restaurant next week to choose between the same two dishes. Today, which dish?

If you are myopic—that is, if you only care about maximizing your immediate gustatory pleasure and not your future self's gustatory pleasure—then you should choose the chicken salad today because it has greater expected utility: its expected utility is 1, while the duck-egg soup's expected utility is 0. And assuming that your information on the two items doesn't change, when you return to the same restaurant next week, you should choose the chicken salad again on the basis of the same expected-utility reasoning.

However, if you also care about your future self's gustatory pleasure, then you should choose the duck-egg soup today. Although choosing the soup doesn't maximize current expected gustatory pleasure, it will give you new information, namely, information about what the duck-egg soup actually tastes like, and this new information is beneficial for promoting your future gustatory pleasure—you can use the new information to make a more informed choice when you return to the same restaurant next week.

Here is a more detailed explanation. If you also care about your future gustatory pleasure, then what you should maximize is expected total-utility, a utility function that factors in not just the gustatory pleasure of your current self but also the

gustatory pleasure of your future selves. For the sake of simplicity, let's assume that this total-utility function is simply the sum of the current and the future utility.[4] The expected total-utility of the chicken salad choice is therefore $1 + 1 = 2$, while the expected total-utility of the duck-egg soup choice is $0.5(5 + 5) + 0.5(-5 + 1) = 3$: if the soup has utility 5, then choosing it today will enable you to learn that it has utility 5, and thus you will choose it again the next time, giving you a 5 in utility again; if it has utility -5, then choosing it today will enable you to learn that it has utility -5, and thus you will avoid the soup the next time and choose the chicken salad instead, giving you a 1 in utility the next time; your confidence in the two possibilities is even. As a result, the duck-egg soup choice has a greater expected total-utility.

The lesson is this: for a person who wants to maximize utility for her future selves as well as utility for her current self, sometimes an option is choiseworthy because it promises to bring new information, and the new information might allow one to maximize expected total-utility even if it doesn't maximize expected current utility.[5]

Now, we want to claim that this lesson generalizes to the decision of whether to change priors. If you only care about the accuracy of your current belief state, then switching priors is a bad idea, since it means adopting a less expectedly accurate credence. But if you are not so myopic, that is, if you also care about the accuracy of your future belief state, then sometimes switching priors is a good idea, since it can bring you new information that you would not otherwise be able to get, and getting new information can lead to a more accurate belief state in the future even from your current perspective.

To drive home the point, consider the following dramatic scenario. Suppose

[4] In a more realistic example, one might discount one's future utility at a certain rate.

[5] This is the main idea behind the exploitation-exploration tradeoff in reinforcement learning (Sutton & Barto, 1998, pp. 26–30).

that your current prior cr at t_0 distributed on $W = \{w_1, w_2\}$ is (1/2, 1/2) and you are considering whether to change it to a different prior (1/3, 2/3). God tells you that if you make the change at t_1 then he will tell you which world is actual, so that you will update to an omniscient credence with the new information at t_2. Suppose that your accuracy measure is given by the Brier score $B(cr, w_i) = -\sum_{w_j}(cr(w_j)–I_{w_i}(w_j))^2$. Then your expected total-accuracy of not changing priors is $-1/2[(1/2-1)^2 + (1/2-0)^2 + (1/2-1)^2 + (1/2-0)^2] - 1/2[(1/2-1)^2 + (1/2-0)^2 + (1/2-1)^2 + (1/2-0)^2] = -1$, whereas the expected total-accuracy of changing priors is $-1/2[(1/3-1)^2 + (2/3-0)^2 + 0] - 1/2[(1/3-0)^2 + (2/3-1)^2 + 0] = -5/9$.

Of course, the above scenario is unrealistic. In real life, you are not guaranteed to get new information by changing priors. But the fact remains that changing priors can sometimes raise the probability of getting new information that you would not otherwise be able to get (we will give some intuitive examples soon). When the expected probability increase is great enough and the new information is substantial enough,[6] the expected total-utility of changing priors—where the total-utility is the sum of the accuracy of your current beliefs and the accuracy of your future beliefs—can be greater than the expected total-utility of maintaining your current priors.

This argument rests on the assumption that changing priors can sometimes raise the probability of gaining new information. This assumption is plausible for two reasons. First, changing priors can help one gain higher-order evidence about one's old priors, specifically, evidence about the irrationality of the old priors. Empirical research on motivated reasoning shows that people are generally better at detecting flaws in reasoning that goes against their beliefs (Kahan et al., 2017). So, by changing priors and reevaluating the old priors from a fresh perspective, one can see more effectively whether the old priors are based on flawed reasoning

[6]Informally, a piece of information is substantial enough relative to a decision problem when it's expected to make a great enough difference to one's decision.

and thus irrational.

Second, changing priors can sometimes promote information gain by providing a greater motivation for conducting inquiry. For instance, sometimes one is neutral on a scientific theory, but one can see that if one becomes highly confident in the theory, it will give one a greater motivation to conduct experiments in order to confirm the belief, and thus one will get information that one would not otherwise be able to get. And in order to be motivated to perform the inquiry and get the new information, one must sometimes actually adopt the different priors, rather than simply assuming or pretending to have them (Aronowitz, 2021). This phenomena of conducting inquiries in order to confirm a belief is not uncommon among scientific communities.[7]

Of course, changing priors doesn't always raise the probability of gaining new significant information. If you know that you won't live long, there isn't much time to gain new information; if you are at the late stage of your inquiry, where relevant information is already abundant, there isn't much new information to be gained; and if you know that all the relevant new information will be generated in an entirely passive manner, namely, in a manner that's entirely insensitive to what you believe, what you believe doesn't affect what new information you gain. So, we are not arguing that it's always rational to change priors.[8]

In sum, the core idea of our argument is this: if you care about the long-term accuracy of your future credences as well as the short-term accuracy of your current

[7] Some have even argued that inquiring in order to confirm a belief is not irrational (Falbo, 2021).

[8] As a reviewer notes, our argument that changing priors can help gain information might apply only to agents who are not ideally rational, and this raises two worries. First, non-ideal agents often don't have direct control over their priors. Second, insofar as we are talking about non-ideal, or 'boundedly rational' agents, why not just say that boundedly rational agents ought to become ideally rational, instead of following these half-measures involving changing priors?

In response to the first worry, we think that rational requirements don't assume 'direct' control: even if one cannot change one's epistemic standards 'at will,' one might still be able to take some intermediary steps to improve them over time. In response to the second worry, we think that theorizing about half-measures can be interesting as 'second-best epistemology': it can be interesting to ask questions like 'given that boundedly rational agents cannot become ideally rational, what's the second-best thing for them to do?'

credences, then your epistemic utility function can be represented by the sum of the two accuracy scores;[9] changing priors can allow you to gain new information and thus increase long-term accuracy; if the expected increase in the long-term accuracy is great enough, it can outweigh the expected loss in the short-term accuracy and thus make changing priors rational.

Now, it's clear that our argument doesn't challenge the three assumptions made in the accuracy argument against changing priors mentioned at the beginning of this paper. Our argument accepts that accuracy is the most important goal in epistemic decisions. It doesn't require us to deviate from the decision rule of maximizing expected utility. It also doesn't challenge the assumption that accuracy measures are strictly proper (as illustrated in the above God case, where we use the Brier score as our accuracy measure.)

Of course, the epistemic total-utility function, which factors in the causal impact of possessing a credence on the accuracy of one's future credence, is not strictly proper. But this doesn't challenge the strict-propriety assumption about accuracy measures, since the total-utility function is not an accuracy measure, although it's entirely based on accuracy considerations. More precisely, it's not the kind of accuracy measures that are at the heart of existing arguments for strict propriety. Those kinds of accuracy measures are concerned with the accuracy of a credence function when we view the credence function as an abstract mathematical entity, without considering whether it's had or who has it; they are not concerned with the causal consequence (in terms of accuracy gain or loss) of having the credence function (Carr, 2017, pp. 519–20).[10] This is understandable: the causal consequence

[9]For ease of exposition, we consider the accuracy scores from only two time slices. Our epistemic total-utility function can allow infinitely many time slices, if the utility of future accuracy is discounted at appropriate rates, so that the sum of the utility of infinitely many time slices is a convergent series.

[10]The main argument for strict propriety is as follows (Joyce, 2009, pp. 277-9; Greaves & Wallace, 2006, p.621). Without strict propriety, some probability functions won't maximize expected accuracy relative to the function itself; such probabilities will be ruled out as irrational a priori, i.e., they will be ruled out regardless of one's evidential situation, because the person who has the probability

of having a mathematical function as one's credence is highly individual, and thus having a credence cannot be given a general accuracy score.

3 Violating Evidentialism?

We've argued that changing priors can promote long-term accuracy by promoting information gain. Now, here is a worry: our argument assumes that we can trade off short-term accuracy for long-term accuracy; such a trade-off can be problematic because it violates evidentialism in some cases: there are cases where a credence doesn't conform with one's current evidence, but adopting it promises to bring valuable information and thus is permissible according to our proposal.[11]

We have two responses to this evidentialist worry. First, this paper only aims to convince epistemic consequentialists, i.e., those who agree that whether changing priors is rational depends on whether it maximizes expected accuracy. It's not intended to convince those who think about the rationality of changing priors from an evidentialist point of view.[12]

function will be self-undermining. But we should not rule out a probability function as irrational a priori, since each probability function may be rational in some evidential situations.

This line of reasoning works only if we consider a probability function as an abstract mathematical function, regardless of whether it's had or who has it as a credence function. If we consider the causal effect of having a probability function for some person at some time, then saying that having a probability function doesn't maximize expected accuracy relative to the function itself for some person at some time won't rule out the function as irrational a priori—it might be still rational for other people, or for the same person in a different situation, to have the probability function.

[11] As a reviewer points out, considering information gain might threaten not just evidentialism but almost all epistemic norms: for almost any plausible epistemic norm, we can imagine a situation where violating the norm can help gain information. We share this concern, and we think that the response developed in this section is applicable to other norms: considering information gain can still be compatible with the dynamic versions of epistemic norms, which claim that, as one's inquiry progresses, one's violation of those norms should be less and less frequent.

That said, we prefer to focus on evidentialism in this section, since gaining information by changing priors is not just theoretically possible but also empirically supported, as we have argued.

[12] Hedden (2015, pp. 475–6) has proposed an evidentialist argument for stable priors appealing to the uniqueness thesis, which says that any evidence supports only one credence function. Given uniqueness, a rational person doesn't change priors because doing so leads to a credence that's unsupported by evidence. This paper doesn't try to engage with this style of argument. For a response to Hedden, see Titelbaum (2015, pp. 673–4).

Second, and more importantly, even from an evidentialist point of view, the kind of tradeoff we advocate here is not egregious, because it's compatible with what can be called 'dynamic evidentialism': although there are times when one must violate evidentialism, one's dynamic epistemic behaviors in an inquiry should exhibit an evidentialist trend in the following sense: as one's inquiry progresses, one's violations of evidentialism will become less and less frequent. This is because as one's inquiry progresses, one gains more and more information, and thus the probability of gaining new valuable information decreases; so, the probability that changing priors promote information gain decreases. This means that, as one's inquiry progresses, one should increasingly focus on 'maximizing expected accuracy given the information one already has' rather than 'seeking new information.' As a result, as one's inquiry progresses, one should behave more and more like an evidentialist.[13]

4 Conclusion

We have argued in this paper that changing priors can sometimes be rational by promoting long-term accuracy. Our argument can be used to illustrate a broader point, namely, that considering long-term accuracy has significant ramifications for the accuracy-first project. This is because taking into account long-term accuracy opens up new possible ways of defining the epistemic utility function, where utility is still entirely based on accuracy considerations. In this paper, we take epistemic utility as a simple sum of short-term and long-term accuracy. However, we can also impose additional structures on the utility function. For example, we may want the utility function to reflect not just whether the total accuracy score is excellent,

[13]This idea is nicely captured by the popular epsilon-greedy algorithm with a decaying epsilon in reinforcement learning, which says that as one's inquiry progresses, the frequency of random exploration decreases and the frequency of exploitation given one's evidence increases (Sutton & Barto, 1998).

but also some global features, such as whether the accuracy of beliefs over time shows an improving trend and whether the improvement is fast enough.

Considering these new structures that we might impose on epistemic utility functions has many benefits. It will not only provide a fresh understanding of our accuracy goals, but also provide new resources to be used in the endeavor of recovering rationality norms from accuracy considerations. As it happens, some fruitful attempts in this direction have already been made in the research on formal learning theory, a research program that aims to recover rationality norms from long-term accuracy goals.[14] So, this paper can also be viewed as a call for further research in this direction.[15]

Bibliography

Aronowitz, S. (2021). Exploring by believing. *Philosophical Review*, *130*(3), 339–383.

Briggs, R. (2009). Distorted reflection. *Philosophical Review*, *118*, 9–85.

Carr, J. R. (2017). Epistemic utility theory and the aim of belief. *Philosophy and Phenomenological Research*, *95*(3), 511–534.

Christensen, D. (1994) Conservatism in epistemology. *Nous*, *28*(1), 69–89.

Falbo, A. (2021). Inquiry and confirmation. *Analysis*, *81*(4), 622–631.

Greaves, H. & D. Wallace (2006). Justifying conditionalization: Conditionalization maximizes expected epistemic utility. *Mind*, *115*(459), 607–632.

Harman, G. (1995). Rationality. In Osherson, D. N. & Smith, E. E. (eds.) *An Invitation to Cognitive Science: Thinking (3)*. Cambridge: MIT Press, 175–212.

[14] A classic example is the learning-theoretic argument against counterinductive priors, which says that such priors are irrational because a counterinductivist cannot converge to the truth in the long run (Schulte, 2018). More examples of learning-theoretic arguments for rationality norms can be found in Ye (2023, pp. 165–6).

[15] An earlier version of this paper was presented at Zhejiang University and Wuhan University. This work has benefited greatly from the thoughtful comments and suggestions provided by the audience.

Hedden, B. (2015). Time-slice rationality. *Mind*, *124*(494), 449–491.

Joyce, J. M. (2009). Accuracy and coherence: Prospects for an alethic epistemology of partial belief. In F. Huber & C. Schmidt-Petri (Eds.), *Degrees of Belief* (V342, pp. 263–297). Springer.

Kahan, D. M., Peters, E., Dawson, E. & Slovic, P. (2017). Motivated numeracy and enlightened self-government. *Behavioural Public Policy, 1*, 54–86.

Meacham, C. J. G. (2014). Impermissive bayesianism. *Erkenntnis, 79*(Suppl 6), 1185–1217.

Meacham, C. J. G. (2015). Understanding conditionalization. *Canadian Journal of Philosophy, 45*(5), 767–797.

Meacham, C. J. G. (2016). Ur-Priors, conditionalization, and ur-prior conditionalization. *Ergo: An Open Access Journal of Philosophy, 3*.

Schoenfield, M. (2014). Permission to believe: Why permissivism is true and what it tells us about irrelevant influences on belief. *Nous, 48*(2), 193–218.

Schoenfield, M. (forthcoming). Meditations on beliefs formed arbitrarily. In T. S. Gendler, J. Hawthorne & J. Chung (eds.), *Oxford Studies in Epistemology, 7.*

Schulte, O. (2018). Formal learning theory. *The Stanford Encyclopedia of Philosophy* (Spring 2018 Edition), E. N. Zalta (ed.), URL = http://plato.stanford.edu/archives/spr2018/entries/formal/.

Sutton, R. S. & Barto, A. G. (1998). *Reinforcement Learning: An Introduction*. Cambridge: The MIT Press.

Titelbaum, M. G. (2015). Continuing on. *Canadian Journal of Philosophy, 45*(5), 670–691.

Vahid, H. (2004). Varieties of epistemic conservatism. *Synthese, 141*(1), 97–122.

Ye, R. (2023). Permissivism, the value of rationality, and a convergence-theoretic epistemology. *Philosophy and Phenomenological Research, 106*, 157–175.