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Similarity Measure of Refined Single-Valued Neutrosophic Sets and Its Multicriteria Decision Making Method

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Abstract. This paper introduces a refined single-valued neutrosophic set (RSVNS) and presents a similarity measure of RSVNSs. Then a multicriteria decision-making method with RSVNS information is developed based on the similarity measure of RSVNSs. By the similarity measure between each alternative and the ideal solution (ideal alternative), all the alternatives can be ranked and the best one can be selected as well. Finally, an actual example on the selecting problems of construction projects demonstrates the application and effectiveness of the proposed method.

Keywords: Refined single-valued neutrosophic set, similarity measure, decision making.

1 Introduction

To deal with indeterminate and inconsistent information, Smarandache [1] proposed a neutrosophic set, which is composed of the neutrosophic components of truth, indeterminacy, and falsity denoted by *T*, *I*, *F*. Then, Wang et al. [2] constrained the neutrosophic set to a single-valued neutrosophic set (SVNS) as a subclass of the neutrosophic set for convenient actual applications. Further, Smarandache [3] extended the classical neutrosophic logic to n-valued refined neutrosophic logic, in which neutrosophic components *T*, *I*, *F* are refined (splitted) into $T_1, T_2, ..., T_p$ and $I_1, I_2, ..., I_r$, and $F_1, F_2, ..., F_t$, respectively, and constructed as a *n*-valued refined neutrosophic set. In existing literature [4-7], neutrosophic refined sets were studied and applied to medical diagnosis and decision making. However, the existing neutrosophic refined set is also a single-valued neutrosophic multiset [6] in the concept. In this paper, we present a refined single-valued neutrosophic set (RSVNS), then its concept is different from the concept of single-valued neutrosophic multisets (neutrosophic refined sets) [4-7]. In fact, RSVNSs are scarcely studied and applied in science and engineering fields. Therefore, it is necessary to propose a similarity measure between RSVNSs and its decision making method in this paper.

The rest of the paper is constructed as follows. Section 2 reviews basic concepts of a SVNS and a neutrosophic refined set (single-valued neutrosophic multiset). Section 3 introduces a RSVNS and a similarity measure of RSVNSs. Section 4 presents a multicriteria decision-making method based on the similarity method under a RSVNS environment. In section 5, an actual example is provided for the decision-making problem of selecting construction projects to illustrate the application of the proposed method. Section 6 contains conclusions and future research.

2 Preliminaries

Definition 1 [2]. Let *U* be a universe of discourse, then a SVNS *A* in *U* is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, such that $T_A(x), I_A(x), F_A(x) \in [0, 1]$ and $0 \le T_A(x) + T_A(x) + T_A(x) \le 3$. Thus, a SVNS *A* can be expressed as $A=\{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in U\}$.

Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universe of discourse, and *A* and *B* be two (non-refined) single-valued neutrosophic sets, $A=\{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle \mid x_i \in U\}$ and $B=\{\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle \mid x_i \in U\}$. Majumdar and Samanta's similarity method of two (non-refined) single-valued neutrosophic sets *A* and *B* is:

$$M_{MS}(A,B)=\frac{\sum_{i=1}^{n}\left\{\begin{array}{l}\min(T_{iA}(x_i),T_{iB}(x_i))\\+\min(I_{iA}(x_i),I_{iB}(x_i))\\+\min(F_{iA}(x_i),F_{iB}(x_i))\end{array}\right\}}{\sum_{i=1}^{n}\left\{\begin{array}{l}\max(T_{iA}(x_i),T_{iB}(x_i))\\+\max(I_{iA}(x_i),I_{iB}(x_i))\\+\max(F_{iA}(x_i),F_{iB}(x_i))\end{array}\right\}}. \quad (1)$$

Based on *n*-valued refined neutrosophic sets [3], Ye and Ye [6] introduced a single-valued neutrosophic multisets (also called a single-valued neutrosophic refined set (SVNRS) [4, 5, 7]) and defined it below.

Definition 2. Let U be a universe of discourse, then a SVNRS R in U can be defined as follows:

$$R=\left\{\left\langle \begin{array}{l} x,(T_{1R}(x),T_{2R}(x),...,T_{pR}(x)),(I_{1R}(x),I_{2R}(x),\\ ...,I_{pR}(x)),(F_{1R}(x),F_{2R}(x),...,F_{pR}(x)) \end{array}\right\rangle \mid x\in U\right\},$$

where p is a positive integer, $T_{1R}(x),T_{2R}(x),...,T_{pR}(x)$ $:U\rightarrow[0,1]$, $I_{1R}(x),I_{2R}(x),...,I_{pR}(x)$ $:U\rightarrow[0,1]$, and $F_{1R}(x),F_{2R}(x),...,F_{pR}(x)$ $:U\rightarrow[0,1]$, and there are $0\le T_{jR}(x)+I_{jR}(x)+F_{jR}(x)\le 3$ for $j = 1, 2, …, p$.

Definition 3. Let two SVNRS R and S in U be:

$$R=\left\{\left\langle \begin{array}{l} x,(T_{1R}(x),T_{2R}(x),...,T_{pR}(x)),(I_{1R}(x),I_{2R}(x),\\ ...,I_{pR}(x)),(F_{1R}(x),F_{2R}(x),...,F_{pR}(x)) \end{array}\right\rangle \mid x\in U\right\},$$

$$S=\left\{\left\langle \begin{array}{l} x,(T_{1S}(x),T_{2S}(x),...,T_{pS}(x)),(I_{1S}(x),I_{2S}(x),\\ ...,I_{pS}(x)),(F_{1S}(x),F_{2S}(x),...,F_{pS}(x)) \end{array}\right\rangle \mid x\in U\right\}.$$

Then there are the following relations of R and S:

(1) Containment:

$R\subseteq S$, if and only if $T_{jR}(x)\le T_{jS}(x)$, $I_{jR}(x)\ge I_{jS}(x)$, $F_{jR}(x)\ge F_{jS}(x)$ for $j = 1, 2, …, p$;

(2) Equality:

$R = S$, if and only if $T_{jR}(x) = T_{jS}(x)$, $I_{jR}(x) = I_{jS}(x)$, $F_{jR}(x) = F_{jS}(x)$ for $j = 1, 2, …, p$;

(3) Union:

$$R\cup S=\left\{\left\langle \begin{array}{l} x,(T_{1R}(x)\vee T_{1S}(x),T_{2R}(x)\vee T_{2S}(x),...,T_{pR}(x)\vee T_{pS}(x)),\\ (I_{1R}(x)\wedge I_{1S}(x),I_{2R}(x)\wedge I_{2S}(x),...,I_{pR}(x)\wedge I_{pS}(x)),\\ (F_{1R}(x)\wedge F_{1S}(x),F_{2R}(x)\wedge F_{2S}(x),...,F_{pR}(x)\wedge F_{pS}(x)) \end{array}\right\rangle \mid x\in U\right\};$$

(4) Intersection:

$$R\cap S=\left\{\left\langle \begin{array}{l} x,(T_{1R}(x)\wedge T_{1S}(x),T_{2R}(x)\wedge T_{2S}(x),\\ ...,T_{pR}(x)\wedge T_{pS}(x)),(I_{1R}(x)\vee I_{1S}(x),\\ I_{2R}(x)\vee I_{2S}(x),...,I_{pR}(x)\vee I_{pS}(x)),\\ (F_{1R}(x)\vee F_{1S}(x),F_{2R}(x)\vee F_{2S}(x),\\ ...,F_{pR}(x)\vee F_{pS}(x)) \end{array}\right\rangle \mid x\in U\right\}.$$

3 Similarity Methods of RSVNSs

In this section, we introduce a RSVNS and propose a similarity method between RSVNSs based on the extension of Majumdar and Samanta's similarity method of two (non-refined) single-valued neutrosophic sets [8].

Definition 4. Let R and S in the universe of discourse $U = \{x_1, x_2, …, x_n\}$ be two refined single-valued neutrosophic sets, which are defined as

$$R=\left\{\left\langle \begin{array}{l} x_i,(T_{1R}(x_i),T_{2R}(x_i),...,T_{p_iR}(x_i)),\\ (I_{1R}(x_i),I_{2R}(x_i),...,I_{p_iR}(x_i)),\\ (F_{1R}(x_i),F_{2R}(x_i),...,F_{p_iR}(x_i)) \end{array}\right\rangle \mid x_i\in U\right\},$$

$$S=\left\{\left\langle \begin{array}{l} x_i,(T_{1S}(x_i),T_{2S}(x_i),...,T_{p_iS}(x_i)),\\ (I_{1S}(x_i),I_{2S}(x_i),...,I_{p_iS}(x_i)),\\ (F_{1S}(x_i),F_{2S}(x_i),...,F_{p_iS}(x_i)) \end{array}\right\rangle \mid x_i\in U\right\},$$

where p_i is a positive integer, and all $T_{jR}(x_i)$, $I_{jR}(x_i)$, $F_{jR}(x_i)$ and $T_{jS}(x_i)$, $I_{jS}(x_i)$, $F_{jS}(x_i)$ (i = 1, 2, …, n; j = 1, 2, …, p_i) belong to [0, 1].

As an extension of Majumdar and Samanta's similarity method of SVNSs [8], we present a similarity method between two RSVNSs R and S as follows:

$$M(R,S)=\frac{\sum_{i=1}^{n}\sum_{j=1}^{p_i}\left\{\begin{array}{l}\min(T_{jR}(x_i),T_{jS}(x_i))\\ +\min(I_{jR}(x_i),I_{jS}(x_i))\\ +\min(F_{jR}(x_i),F_{jS}(x_i))\end{array}\right\}}{\sum_{i=1}^{n}\sum_{j=1}^{p_i}\left\{\begin{array}{l}\max(T_{jR}(x_i),T_{jS}(x_i))\\ +\max(I_{jR}(x_i),I_{jS}(x_i))\\ +\max(F_{jR}(x_i),F_{jS}(x_i))\end{array}\right\}}. \quad (2)$$

Obviously, the above similarity measure $M(R, S)$ satisfies the following properties:

(1) $0\le M(R, S)\le 1$;

(2) $M(R, S) = M(S, R)$

(3) $M(R, S) = 1$ if and only if $R = S$.

In general, we usually consider the weights of criteria. Assume that the weight of each criterion x_i is w_i (i = 1, 2, …, n), with $w_i\in[0, 1]$ and $\sum_{i=1}^{n}w_i=1$. Then, we can introduce the weighted similarity formula:

$$M_w(R,S)=\frac{\sum_{i=1}^{n}w_i\sum_{j=1}^{p_i}\begin{Bmatrix}\min(T_{jR}(x_i),T_{jS}(x_i))\\+\min(I_{jR}(x_i),I_{jS}(x_i))\\+\min(F_{jR}(x_i),F_{jS}(x_i))\end{Bmatrix}}{\sum_{i=1}^{n}w_i\sum_{j=1}^{p_i}\begin{Bmatrix}\max(T_{jR}(x_i),T_{jS}(x_i))\\+\max(I_{jR}(x_i),I_{jS}(x_i))\\+\max(F_{jR}(x_i),F_{jS}(x_i))\end{Bmatrix}}. \quad (3)$$

4 Decision-making method using the similarity measure

In a decision making problem, there is a set of alternatives $R = \{R_1, R_2, \ldots, R_m\}$, which needs to satisfies a set of criteria $C = \{C_1, C_2, \ldots, C_n\}$, where C_i ($i = 1, 2, \ldots, n$) may be splitted into some sub-criteria C_{ij} ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, p_i$). If the decision maker provides the suitability evaluation values of the criteria for C_i (i =1,2,..., n) on the alternative R_k (k = 1, 2,…, m) by using a RSVNS:

$$R_k=\left\{\left\langle\begin{matrix}C_i,(T_{1R_k}(C_i),T_{2R_k}(C_i),\ldots,T_{p_iR_k}(C_i)),\\(I_{1R_k}(C_i),I_{2R_k}(C_i),\ldots,I_{p_iR_k}(C_i)),\\(F_{1R_k}(C_i),F_{2R_k}(C_i),\ldots,F_{p_iR_k}(C_i))\end{matrix}\right\rangle \mid C_i\in C\right\}.$$

Then for convenience, each basic element in the RSVNS R_k is represented by the refined single-valued neutrosophic number (RSVNN):

$$\left\langle(T_{1R_k},T_{2R_k},\ldots,T_{p_iR_k}),(I_{1R_k},I_{2R_k},\ldots,I_{p_iR_k}),(F_{1R_k},F_{2R_k},\ldots,F_{p_iR_k})\right\rangle$$

for $i = 1, 2, \ldots, n$; $k = 1, 2, \ldots, m$. Hence, we can construct the refined single-valued neutrosophic decision matrix D, as shown in Table 1.

When the weights of criteria are considered as the different importance of each criterion C_i ($i = 1, 2, \ldots, n$), the weight vector of the three criteria is given by $W = (w_1, w_2, \ldots, w_n)$ with $w_i \geq 0$ and $\sum_{i=1}^{n} w_i = 1$. Thus, the decision-making steps are described as follows:

Step 1: Based on the refined single-valued neutrosophic decision matrix D, we can determine the ideal solution (ideal RSVNN) by

$$s_i^*=\left\langle\begin{matrix}(\max_k(T_{1R_k}),\max_k(T_{2R_k}),\ldots,\max_k(T_{p_iR_k})),\\(\min_k(I_{1R_k}),\min_k(I_{2R_k}),\ldots,\min_k(I_{p_iR_k})),\\(\min_k(F_{1R_k}),\min_k(F_{2R_k}),\ldots,\min_k(F_{p_iR_k}))\end{matrix}\right\rangle, \quad (4)$$

which is constructed as the ideal alternative $S^* = \{s_1^*, s_2^*, \ldots, s_n^*\}$.

Step 2: The similarity measure between each alternative R_k ($k = 1, 2, \ldots, m$) and the ideal alternative S^* can be calculated according to Eq. (3) and the values of $M_w(R_k, S^*)$ for $k = 1, 2, \ldots, m$ can be obtained.

Step 3: The alternatives are ranked in a descending order according to the values of $M_w(R_k, S^*)$ for $k = 1, 2, \ldots, m$. The greater value of $M_w(R_k, S^*)$ means the better alternative R_k.

Step 4: End.

5 Actual example on the selection of construction projects

In this section, we give the application of the decision making method for the selection of construction projects.

A construction company needs to determine the selecting problem of construction projects. Decision makers provide four construction projects as a set of four alternatives $R = \{R_1, R_2, R_3, R_4\}$. Then, the selection of these construction projects is dependent on three main criteria and seven sub-criteria: (1) Financial state (C_1): budget control (C_{11}) and risk/return ratio (C_{12}); (2) Environmental protection (C_2): public relation (C_{21}), geographical location (C_{22}), and health and safety (C_{23}); (3) Technology (C_3): technical knowhow (C_{31}), technological capability (C_{32}).

Experts or decision makers are required to evaluate the four possible alternatives under the above three criteria (seven sub-criteria) by suitability judgments, which are represented by RSVNSs. Thus we can construct the following refined single-valued neutrosophic decision matrix D, as shown in Table 2.

Table 1. The refined single-valued neutrosophic decision matrix D

	$C_1(C_{11},C_{12},\ldots,C_{1p_1})$	…	$C_n(C_{n1},C_{n2},\ldots,C_{np_n})$
R_1	$\langle(T_{1R_1},T_{2R_1},\ldots,T_{p_1R_1}),(I_{1R_1},I_{2R_1},\ldots,I_{p_1R_1}),(F_{1R_1},F_{2R_1},\ldots,F_{p_1R_1})\rangle$	…	$\langle(T_{1R_1},T_{2R_1},\ldots,T_{p_nR_1}),(I_{1R_1},I_{2R_1},\ldots,I_{p_nR_1}),(F_{1R_1},F_{2R_1},\ldots,F_{p_nR_1})\rangle$
R_2	$\langle(T_{1R_2},T_{2R_2},\ldots,T_{p_1R_2}),(I_{1R_2},I_{2R_2},\ldots,I_{p_1R_2}),(F_{1R_2},F_{2R_2},\ldots,F_{p_1R_2})\rangle$	...	$\langle(T_{1R_2},T_{2R_2},\ldots,T_{p_nR_2}),(I_{1R_2},I_{2R_2},\ldots,I_{p_nR_2}),(F_{1R_2},F_{2R_2},\ldots,F_{p_nR_2})\rangle$
…	…	...	…
R_m	$\langle(T_{1R_m},T_{2R_m},\ldots,T_{p_1R_m}),(I_{1R_m},I_{2R_m},\ldots,I_{p_1R_m}),(F_{1R_m},F_{2R_m},\ldots,F_{p_1R_m})\rangle$	...	$\langle(T_{1R_m},T_{2R_m},\ldots,T_{p_nR_m}),(I_{1R_m},I_{2R_m},\ldots,I_{p_nR_m}),(F_{1R_m},F_{2R_m},\ldots,F_{p_nR_m})\rangle$

Table 2. Defined single-valued neutrosophic decision matrix *D* for the four alternatives on three criteria (seven sub-criteria)

	C_1 (C_{11}, C_{12})	C_2 (C_{21}, C_{22}, C_{23})	C_3 (C_{31}, C_{32})
R_1	<(0.6, 0.7), (0.2, 0.1), (0.2, 0.3)>	<(0.9, 0.7, 0.8), (0.1, 0.3, 0.2), (0.2, 0.2, 0.1)>	<(0.6, 0.8), (0.3, 0.2), (0.3, 0.4)>
R_2	<(0.8, 0.7), (0.1, 0.2), (0.3, 0.2)>	<(0.7, 0.8, 0.7), (0.2, 0.4, 0.3), (0.1, 0.2, 0.1)>	<(0.8, 0.8), (0.1, 0.2), (0.1, 0.2)>
R_3	<(0.6, 0.8), (0.1, 0.3), (0.3, 0.4)>	<(0.8, 0.6, 0.7), (0.3, 0.1, 0.1), (0.2, 0.1, 0.2)>	<(0.8, 0.7), (0.4, 0.3), (0.2, 0.1)>
R_4	<(0.7, 0.6), (0.1, 0.2), (0.2, 0.3)>	<(0.7, 0.8, 0.7), (0.2, 0.2, 0.1), (0.1, 0.2, 0.2)>	<(0.7, 0.7), (0.2, 0.3), (0.2, 0.3)>

Then, the weight vector of the three criteria is given by W = (0.4, 0.3, 0.3). Thus, the proposed decision making method is applied to the selecting problem of the construction projects. Consequently, the decision-making steps are described as follows:

Step 1: By Eq. (4), the ideal solution (ideal RSVNS) can be determined as the following ideal alternative:

S^* ={<(0.8, 0.8), (0.1, 0.1), (0.2, 0.2)>, <(0.9, 0.8, 0.8), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1)>, <(0.8, 0.8), (0.1, 0.2), (0.1, 0.1)>}.

Step 2: According to Eq. (3), the weighted similarity measure values between each alternative R_k (k = 1, 2, 3, 4) and the ideal alternative S^* can be obtained as follows:

$M_w(R_1, S^*) = 0.7743$, $M_w(R_2, S^*) = 0.8370$, $M_w(R_3, S^*) = 0.7595$, and $M_w(R_4, S^*) = 0.7778$.

Step 3: Since the measure values are $M_w(R_2, S^*) > M_w(R_4, S^*) > M_w(R_1, S^*) > M_w(R_3, S^*)$, the ranking order of the four alternatives is $R_2 > R_4 > R_1 > R_3$. Hence, the alternative R_2 is the best choice among all the construction projects.

6 Conclusion

This paper introduced RSVNSs and presented the similarity measure of RSVNSs. Then, we proposed a similarity measure-based multicriteria decision-making method under a RSVNS environment. In the decision-making process, through the similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be selected as well. Finally, an actual example on the selecting problem of construction projects demonstrated the application of the proposed method. The main advantage of the proposed approach is easy evaluation and more suitable for actual applications in decision-making problems with RSVNS information. In the future, we shall extend the proposed decision-making method to medical diagnosis and fault diagnosis.

References

[1] F. Smarandache. Neutrosophy: Neutrosophic probability, set, and logic, American Research Press, Rehoboth, USA, 1998.

[2] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman. Single valued neutrosophic, sets. Multispace and Multi structure, 4 (2010), 410–413.

[3] F. Smarandache. n-Valued refined neutrosophic logic and its applications in physics. Progress in Physics, 4 (2013), 143-146.

[4] S. Broumi and F. Smarandache. Neutrosophic refined similarity measure based on cosine function. Neutrosophic Sets and Systems, 6 (2014), 42-48.

[5] S. Broumi and I. Deli. Correlation measure for neutrosophic refined sets and its application in medical diagnosis, Palestine Journal of Mathematics, 3(1) (2014), 11-19.

[6] S. Ye and J. Ye. Dice similarity measure between single valued neutrosophic multisets and its application in medical diagnosis. Neutrosophic sets and System, 6 (2014), 49-54.

[7] K. Mondal and S. Pramanik. Neutrosophic refined similarity measure based on cotangent function and its application to multi-attribute decision making, Global Journal of Advanced Research, 2(2) (2015), 486-496.

[8] P. Majumdar, and S. K. Samanta. On similarity and entropy of neutrosophic sets. Journal of Intelligent and Fuzzy Systems, 26 (2014), 1245-1252.

Received: February 10, 2016. Accepted: June 23, 2016.